The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **Spring Semester 2017–2018**

Vectors and Mechanics **2 hours**

Attempt all the questions. The allocation of marks is shown in brackets. The total number of marks available is 60. All vector quantities must be underlined. Use $g = 9.8\ ms^{-2}$ when needed.

1 If $\boldsymbol{a}$ represents a velocity of 2 ms^{-1} east and $\boldsymbol{b}$ represents a velocity of $2\sqrt{2}$ ms^{-1} north-west, determine the velocities represented by

(i) $-2\boldsymbol{b}$,

(ii) $\boldsymbol{a} + \boldsymbol{b}$. ***(2 marks)***

2 The position vectors of points A and B relative to the origin O are $\overrightarrow{OA} = 2\boldsymbol{i} + \boldsymbol{j}$ and $\overrightarrow{OB} = -\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$, respectively. Find the position vector of a point P relative to the origin O such that it has unit length and is in the direction of $\overrightarrow{AB}$.

(3 marks)

3 The distinct points A, B and C lie on a circle centred at the origin O. The points AOC lie on the same diameter of the circle and point B does not. Let $\boldsymbol{a} = \overrightarrow{OA}$ and $\boldsymbol{b} = \overrightarrow{OB}$.

(i) Express $\overrightarrow{BA}$ and $\overrightarrow{BC}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

(ii) Calculate $\overrightarrow{BA} \cdot \overrightarrow{BC}$. What does this tell us about angle ABC?

(5 marks)

4 Simplify

(i) $(\boldsymbol{a} + 3\boldsymbol{b}) \times (\boldsymbol{a} - \boldsymbol{b})$,

(ii) $\boldsymbol{a} \cdot [\boldsymbol{a} \times (\boldsymbol{a} + \boldsymbol{b})]$.

(2 marks)

5 Calculate the shortest distance between the two lines $\boldsymbol{r}_1 = \boldsymbol{i} + 5\boldsymbol{j} - 4\boldsymbol{k} + \lambda(3\boldsymbol{i} - 3\boldsymbol{j} + \boldsymbol{k})$ and $\boldsymbol{r}_2 = -2\boldsymbol{i} + \boldsymbol{j} - 9\boldsymbol{k} + \mu(4\boldsymbol{i} + 9\boldsymbol{j} + 2\boldsymbol{k})$, where λ and μ are parameters. Give your answer to an accuracy of two decimal places. ***(5 marks)***

6 Let $\boldsymbol{a} = 5t^2\boldsymbol{i} + t\boldsymbol{j} - t^3\boldsymbol{k}$ and $\boldsymbol{b} = \sin(t)\boldsymbol{i} - \cos(t)\boldsymbol{j}$. Calculate

(i) $\dfrac{d}{dt}(\boldsymbol{a} \cdot \boldsymbol{b})$,

(ii) $\dfrac{d}{dt}(\boldsymbol{a} \times \boldsymbol{b})$.

(6 marks)

7 Points O, A, B and C lie on a straight line in this order with $AB = 6$ m and $BC = 10$ m. An object starts from rest at O and moves along this line with a constant acceleration passing through points A, B and C. At B its speed is 4 m s^{-1} and at C its speed is 6 m s^{-1}. What is its speed at A? ***(3 marks)***

8 (i) A point moves along a straight line with acceleration which is proportional to its velocity. Find its velocity and displacement if their initial values are respectively $v_0 > 0$ and x_0 at $t = 0$. ***(10 marks)***

(ii) Find the general solution of

$$\frac{d^2x}{dt^2} = \left(\frac{dx}{dt}\right)^2 \mathrm{e}^t.$$

(5 marks)

9 A car is approaching a bend which is of radius 40 m. The coefficient of friction between the road and the car's tyres is 0.4. Find the maximum speed at which the car an negotiate the bend without skidding. ***(6 marks)***

10 A conical pendulum has a string of length 3m. If the pendulum makes 1 revolution per second, find the angle in radians the string makes with the vertical.

(4 marks)

11 (i) A lift is taking some students from the ground floor to the 7^{th} floor. The students and the lift have a combined mass of 1400 kg. If the lift accelerated upwards at 1.4 m s^{-2}, what is the tension in the lift cable? ***(5 marks)***

(ii) Draw a diagram to show the forces acting on a particle when it is pulled across a rough horizontal surface by a string at an angle of $\dfrac{\pi}{6}$ to the horizontal. ***(4 marks)***

End of Question Paper