Data provided: Formula sheet

MAS153/MAS159

The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **Spring Semester 2017–2018**

Mathematics (Materials) **3 hours**
Mathematics For Chemists

All questions are compulsory. The marks awarded to each question or section of question are shown in italics.

1 Find all solutions of the equation

$$\sin x \cos x = \frac{1}{4}.$$ *(5 marks)*

2 Solve the equation

$$\ln(x+1)^2 = \ln(x-2)+\ln(x-5).$$ *(6 marks)*

3 Identify the number d such that

$$3^d = \frac{9 \times 2^{\frac{2}{3}} \times 14 \times 3^{\frac{2}{3}}}{2 \times 6^{\frac{4}{6}} \times 7}.$$ *(4 marks)*

4 Simplify

$$\frac{(64x^3y^6z^3)^{\frac{1}{3}}}{(\sqrt{2}x^{\frac{1}{4}}y\sqrt{z})^2}.$$ *(4 marks)*

5 Factorise $2x^2 - 10x + 12$. *(3 marks)*

6 Given a function $y = f(x)$ where $f(x) = e^{\frac{1}{x}}$.

(i) Draw the graph of the function $y = f(x)$. ***(4 marks)***

(ii) Find the domain D and the range R of the function $y = f(x)$. ***(2 marks)***

(iii) Find the inverse $f^{-1}(x)$ of the function $f(x)$. ***(4 marks)***

(iv) Find the domain D' and the range R' of the inverse function $y = f^{-1}(x)$. ***(2 marks)***

7 Solve the following equation for real x:

$$49^x - 6 \times 7^x - 7 = 0. \qquad \textit{(5 marks)}$$

8 Let $y = \ln(\sin(x))$. Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$. ***(5 marks)***

9 Verify the following identity

$$\frac{\cos^4 x + \frac{1}{2}\sin^2(2x) + \sin^4 x}{1 + \tan^2 x} = \cos^2 x. \qquad \textit{(5 marks)}$$

10 (i) Showing your working clearly, find the coefficient of x^4 in the expansion of $(1 + x)^{10}$. ***(2 marks)***

(ii) Use the binomial theorem to evaluate

$$\lim_{x \to \infty} (\sqrt{x^2 + 10x + 2} - x + 5). \qquad \textit{(4 marks)}$$

11 (i) Are the vectors $\mathbf{u} = (2, 0, 1)$ and $\mathbf{v} = (1, 13, -2)$ perpendicular or not? ***(3 marks)***

(ii) A plane passes through the points $\mathbf{a} = (1, 2, 1)$, $\mathbf{b} = (2, 3, -1)$ and $\mathbf{c} = (3, 0, 1)$. Find the Cartesian equation of the plane. ***(6 marks)***

12 Prove, from the definitions of $\sinh x$ and $\cosh x$, the identity

$$\cosh^2(x) - \sinh^2(x) = 1. \qquad \textit{(3 marks)}$$

13 Evaluate

$$\int 3x^2 \ln(x)\, dx.$$ *(4 marks)*

14 Evaluate

$$\int \frac{2t+5}{\sqrt{t^2+2t+5}}\, dt.$$ *(8 marks)*

15 Find the Maclaurin series for $f(x) = e^{\sin x}$, as far as the term in x^3. *(7 marks)*

16 Complex numbers z_1 and z_2 are defined by

$$z_1 = 1 - i, \qquad z_2 = 2 + i.$$

Find, in the form $a + bi$ where a and b are real,

(i) z_1^3, *(2 marks)*

(ii) $\dfrac{z_2}{2z_1 + z_2}$. *(3 marks)*

17 Solve the system of linear equations

$$\begin{cases} x + y + 2z = 2, \\ x + 2y + 3z = 3, \\ 2x + 3y + 5z = 5. \end{cases}$$ *(5 marks)*

18 Find the volume V of the parallelepiped determined by the vectors $\mathbf{a} = (-1, -1, -1)$, $\mathbf{b} = (3, 2, 1)$ and $\mathbf{c} = (2, 1, 1)$. *(4 marks)*

End of Question Paper