1. Find all solutions of the equation
   \[ \sin x \cos x = \frac{1}{4}. \]  
   \(5 \text{ marks}\)

2. Solve the equation
   \[ \ln(x+1)^2 = \ln(x-2) + \ln(x-5). \]  
   \(6 \text{ marks}\)

3. Identify the number \(d\) such that
   \[ 3^d = \frac{9 \times 2^2 \times 14 \times 3^2}{2 \times 6^2 \times 7}. \]  
   \(4 \text{ marks}\)

4. Simplify
   \[ \frac{(64x^3y^6z^3)^{\frac{1}{3}}}{(\sqrt{2x^2}y\sqrt{z})^2}. \]  
   \(4 \text{ marks}\)

5. Factorise \(2x^2 - 10x + 12.\)  
   \(3 \text{ marks}\)
Given a function \( y = f(x) \) where \( f(x) = e^{\frac{1}{2}x} \).

(i) Draw the graph of the function \( y = f(x) \). \((4 \text{ marks})\)

(ii) Find the domain \( D \) and the range \( R \) of the function \( y = f(x) \). \((2 \text{ marks})\)

(iii) Find the inverse \( f^{-1}(x) \) of the function \( f(x) \). \((4 \text{ marks})\)

(iv) Find the domain \( D' \) and the range \( R' \) of the inverse function \( y = f^{-1}(x) \). \((2 \text{ marks})\)

Solve the following equation for real \( x \):

\[ 49^x - 6 \times 7^x - 7 = 0. \] \((5 \text{ marks})\)

Let \( y = \ln(\sin(x)) \). Find \( \frac{dy}{dx} \) and \( \frac{d^2y}{dx^2} \). \((5 \text{ marks})\)

Verify the following identity

\[ \frac{\cos^4 x + \frac{1}{2} \sin^2(2x) + \sin^4 x}{1 + \tan^2 x} = \cos^2 x. \] \((5 \text{ marks})\)

(i) Showing your working clearly, find the coefficient of \( x^4 \) in the expansion of \((1 + x)^{10}\). \((2 \text{ marks})\)

(ii) Use the binomial theorem to evaluate

\[ \lim_{x \to \infty} (\sqrt{x^2 + 10x + 2} - x + 5). \] \((4 \text{ marks})\)

(i) Are the vectors \( \mathbf{u} = (2, 0, 1) \) and \( \mathbf{v} = (1, 13, -2) \) perpendicular or not? \((3 \text{ marks})\)

(ii) A plane passes through the points \( \mathbf{a} = (1, 2, 1), \mathbf{b} = (2, 3, -1) \) and \( \mathbf{c} = (3, 0, 1) \). Find the Cartesian equation of the plane. \((6 \text{ marks})\)

Prove, from the definitions of \( \sinh x \) and \( \cosh x \), the identity

\[ \cosh^2(x) - \sinh^2(x) = 1. \] \((3 \text{ marks})\)
13 Evaluate
\[ \int 3x^2 \ln(x) \, dx. \]  
(4 marks)

14 Evaluate
\[ \int \frac{2t + 5}{\sqrt{t^2 + 2t + 5}} \, dt. \]  
(8 marks)

15 Find the Maclaurin series for \( f(x) = e^{\sin x} \), as far as the term in \( x^3 \).  
(7 marks)

16 Complex numbers \( z_1 \) and \( z_2 \) are defined by
\[ z_1 = 1 - i, \quad z_2 = 2 + i. \]

Find, in the form \( a + bi \) where \( a \) and \( b \) are real,

(i) \( z_1^3 \),  
(2 marks)

(ii) \( \frac{z_2}{2z_1 + z_2} \).  
(3 marks)

17 Solve the system of linear equations
\[ \begin{cases} 
  x + y + 2z = 2, \\
  x + 2y + 3z = 3, \\
  2x + 3y + 5z = 5.
\end{cases} \]  
(5 marks)

18 Find the volume \( V \) of the parallelepiped determined by the vectors \( a = (-1, -1, -1) \), \( b = (3, 2, 1) \) and \( c = (2, 1, 1) \).  
(4 marks)

End of Question Paper