



The
University
Of
Sheffield.

MAS157

SCHOOL OF MATHEMATICS AND STATISTICS

**Spring Semester
2017–2018**

Mathematics For Chemists

2 hours

All questions are compulsory. The marks awarded to each question or section of question are shown in italics.

1 Let $y = \ln(\sin(x))$. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. *(7 marks)*

2 Verify the following identity

$$\frac{\cos^4 x + \frac{1}{2} \sin^2(2x) + \sin^4 x}{1 + \tan^2 x} = \cos^2 x. \quad \text{(9 marks)}$$

3 (a) Showing your working clearly, find the coefficient of x^4 in the expansion of $(1+x)^{10}$. *(3 marks)*

(b) Use the binomial theorem to evaluate

$$\lim_{x \rightarrow \infty} (\sqrt{x^2 + 10x + 2} - x + 5). \quad \text{(5 marks)}$$

4 (a) Are the vectors $\mathbf{u} = (2, 0, 1)$ and $\mathbf{v} = (1, 13, -2)$ perpendicular or not? *(3 marks)*

(b) A plane passes through the points $\mathbf{a} = (1, 2, 1)$, $\mathbf{b} = (2, 3, -1)$ and $\mathbf{c} = (3, 0, 1)$. Find the Cartesian equation of the plane. *(11 marks)*

- 5 Prove, from the definitions of $\sinh x$ and $\cosh x$, the identity

$$\cosh^2(x) - \sinh^2(x) = 1. \quad (5 \text{ marks})$$

- 6 Evaluate

$$\int 3x^2 \ln(x) dx. \quad (8 \text{ marks})$$

- 7 Evaluate

$$\int \frac{2t + 5}{\sqrt{t^2 + 2t + 5}} dt. \quad (13 \text{ marks})$$

- 8 Find the Maclaurin series for $f(x) = e^{\sin x}$, as far as the term in x^3 . (10 marks)

- 9 Complex numbers z_1 and z_2 are defined by

$$z_1 = 1 - i, \quad z_2 = 2 + i.$$

Find, in the form $a + bi$ where a and b are real,

(a) z_1^3 , (4 marks)

(b) $\frac{z_2}{2z_1 + z_2}$. (5 marks)

- 10 Solve the system of linear equations

$$\begin{cases} x + y + 2z = 2, \\ x + 2y + 3z = 3, \\ 2x + 3y + 5z = 5. \end{cases} \quad (9 \text{ marks})$$

- 11 Find the volume V of the parallelepiped determined by the vectors $\mathbf{a} = (-1, -1, -1)$, $\mathbf{b} = (3, 2, 1)$ and $\mathbf{c} = (2, 1, 1)$. (8 marks)

End of Question Paper