The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **Spring Semester 2017–2018**

History of Mathematics **2 hours 30 minutes**

Answer Question 1 and three other questions. If you answer more than three of the Questions 2 to 5 only your best three will be counted.

1 Attempt *three* of questions **(a)**, **(b)**, **(c)**, **(d)** below. If you attempt *more* than three only your best three will be counted.

(a) Below is a translation of **Problem 5** on the Babylonian clay tablet **BM 13901**.

Problem: *Add the area and four thirds of the side of the square*: 0 ; 55. *What is the side*?

Solution: *Four thirds* 1 ; 20. *Halve* 0 ; 40. *Square* 0 ; 26, 40. *Add on* 0 ; 55 ; 1 ; 21, 40. *Square root* 1 ; 10. *Subtract* 0 ; 40 (*that was squared*), 0 ; 30 *is the side.*

Write each of the *seven* different sexagesimal numbers in the extract in the form m/n, where m and n are positive integers. Explain the extract, commenting on aspects of Babylonian mathematics that it illustrates. Use it to write down the *positive* solution of the quadratic equation $x^2 + bx = c$ for positive b and c. ***(7 marks)***

(b) Which proposition in the *Elements* states: *Prime numbers are more than any assigned set of primes*? How is this result expressed in modern texts? Why do you think that Euclid chose the particular way that *he* did? In a sentence, indicate how Euclid showed that every *finite* set of primes can *generate* primes outside itself.

- **(i)** Find two primes under 20 that *generate* the *first four primes* $2, 3, 5, 7$.
- **(ii)** Find the prime(s) *generated* by the *first five primes* $2, 3, 5, 7, 11$. ***(7 marks)***

(c) Write a paragraph on Thomas Harriot's connections with each of the following: the Duke of Northumberland, astronomical observations, and algebra. ***(7 marks)***

(d) Justify, through suitably chosen examples, the following assertion:

In the three decades before Newton's *October 1666 Tract*, mathematicians from England, France and Italy introduced new curves, and tackled problems concerning tangents, areas and arc lengths. Why are *none* of them accorded the title *inventor of the calculus?*
[Dates and detailed arguments are not expected.] ***(7 marks)***

2 Why did Alexander Henry Rhind spend his later years in Egypt? Name the *two* mathematical documents that he purchased in Luxor in 1858. What became of them after his death? Describe their physical appearance. How did Egyptologists overcome the problems facing them when they came to decipher the documents? Incorporate the names Champollion, Eisenlohr, Glanville, Hall, Scott into your answer. ***(11 marks)***

Given that one of the sums below appears on *one* of the documents, the other on the *other*, find which is which. Where, *precisely*, can the *smaller* of the two be found?

$$\frac{1}{25}\,\frac{1}{15}\,\frac{1}{75}\,\frac{1}{200} \quad \& \quad \frac{1}{24}\,\frac{1}{58}\,\frac{1}{174}\,\frac{1}{232}\,.$$ ***(5 marks)***

3 What are *the three classical problems of antiquity*? ***(3 marks)***

(a) Who introduced the *conic sections* into mathematics, and for what purpose? Define these curves in terms intelligible to Greek mathematicians. Which *pairs* of the sections were used to solve *one* of the classical problems? ***(4 marks)***

(b) Who introduced into mathematics the first curve beyond the line and the circle, and for what purpose? Name the curve and describe its construction. ***(4 marks)***

(c) Consider a square of side $2a$, its circumcircle, and the four semicircular arcs of radius a attached to each side of the square and external to it. Find the radius of the circumcircle. Show that the area of a *minor* segment of the circle cut off by a side of the square is $\frac{1}{2}(\pi - 2)a^2$. Deduce that a *lune* bounded by the circle and one of the semicircular arcs can be *squared*. Which Greek mathematician squared such lunes, and for what purpose? ***(5 marks)***

4 This question concerns *algebraic* solutions to the cubic equation $x^3 + px = q$ $(p,\, q > 0)$. Who, *in order*, were the first *five* mathematicians to know of such solutions? How did mathematicians at the time refer to this type of cubic equation? Identify the verse below, giving the title of the book from which it is translated, its author, title, and the year of publication. Why was it composed? What role did it play in the history of the cubic equation?

When the cube and the things together
Are equal to some discrete number,
Find two numbers differing in this one.
Then you will keep this as a habit
That their product should always be equal
Exactly to the cube of a third of the things.
The remainder then as a general rule
Of their cube roots subtracted
Will be equal to your principal thing. ***(6 marks)***

Use the verse to show that one solution of $x^3 + 3ax = 2b$ $(a,\, b > 0)$ is

$$\sqrt[3]{\sqrt{a^3+b^2} + b} \; - \; \sqrt[3]{\sqrt{a^3+b^2} - b}\,.$$ ***(6 marks)***

Deduce that one solution of $x^3 + 3mx = m^2 - m$ $(m > 0)$ is $m^{2/3} - m^{1/3}$. ***(3 marks)***

Write down, in terms of p and q, one solution of $x^3 + px = q$ $(p, q > 0)$. ***(1 mark)***

5 Use the following two propositions from Book I of Archimedes' *On the Sphere and the Cylinder* to express the surface area S, and the volume V of a sphere in terms of its radius r. [You may assume results for the area of a circle in terms of its radius, and the volume of a cone in terms of its height and base radius.]

Proposition 33 The surface area of a sphere is *four* times the greatest circle in it.

Proposition 34 The volume of a sphere is *four* times the cone which has as its base the greatest circle in it and its height equal to the radius of the sphere. ***(2 marks)***

What design did Archimedes request to adorn his tombstone? State and verify the result underlying his request. Who currently employs a logo based on this design? ***(5 marks)***

In **Proposition 2** of his *Method*, Archimedes considers a cylinder with base radius $2r$, and axis the line segment from $(0, 0, 0)$ to $(2r, 0, 0)$. He shows that a sphere of radius r and a cone with both base radius and height equal to $2r$ will, when suspended together from their *centroids* at the point $(-2r, 0, 0)$, balance the cylinder *in situ* about a fulcrum at $(0, 0, 0)$. Use this result to express the volume of a sphere in terms of its radius r.

(6 marks)

How is Archimedes *thought* to have derived $S = 4\pi r^2$ *from* $V = \frac{4}{3}\pi r^3$? What is of interest here concerning the chronology of these two results? ***(3 marks)***

End of Question Paper