



The
University
Of
Sheffield.

MAS5050

SCHOOL OF MATHEMATICS AND STATISTICS

**Spring Semester
2017–2018**

Mathematical Methods for Statistics

2 hours

RESTRICTED OPEN BOOK EXAMINATION

Candidates may bring to the examination lecture notes and associated lecture material (including set textbooks) plus a calculator that conforms to University regulations.

*Candidates should attempt **ALL** questions.*

The paper will be marked out of 80 and the allocation of marks is shown in brackets.

**Please leave this exam paper on your desk
Do not remove it from the hall**

Registration number from U-Card (9 digits)
to be completed by student

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- 1** (i) Evaluate

$$\sum_{n=1}^{11} (1.2^n + 1.2n)$$

giving your answer to two significant figures. **(5 marks)**

- (ii) The 1st, 2nd and 3rd terms of an arithmetic series are a , b and a^2 , where a is a negative number.

The 1st, 2nd and 3rd terms of a geometric series are a , a^2 and b .

- (a) Find the values of a and b . **(3 marks)**

- (b) Find the sum to infinity of the geometric series. **(2 marks)**

- 2** Use Gaussian elimination to solve the following system of equations:

$$x - 4y + z = 10$$

$$2x + y + 3z = -1$$

$$6x + 7y - 2z = 22$$

(10 marks)

- 3** Compute the derivatives of the following functions with respect to x .

- (i) $r(x) = \tan(3x^4 + 1)$ **(3 marks)**

- (ii) $s(x) = \frac{2}{x} - 3 \ln \frac{x^2}{3}$ **(4 marks)**

- (iii) $t(x) = \exp(\sin x)$ **(3 marks)**

- 4** (i) By making a suitable substitution determine the indefinite integral

$$\int 18x^2(6x^3 + 5)^{\frac{1}{4}} dx.$$

(5 marks)

- (ii) By making a suitable substitution compute the definite integral

$$\int_{\pi}^{\frac{3\pi}{2}} (\sin x + 1)^{\frac{1}{2}} \cos x dx.$$

(5 marks)

5 (i) Let

$$A = \begin{pmatrix} -3 & 10 \\ 6 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 7 & -1 \\ 3 & 5 \end{pmatrix}.$$

(a) Calculate $AB + BA$.

(b) Find A^{-1} . (5 marks)

(ii) Find the relationship between the constants a , b and c for which the expression

$$u = \cos(at) \sin(bx)$$

satisfies the equation

$$\frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}.$$

(5 marks)

6 Let

$$A = \begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}.$$

Find the eigenvalues of A and their associated eigenvectors. (10 marks)

7 (i) Let S be the region bounded by the curves $y = \sqrt{x}$ and $y = x^3$. Sketch the region denoted by S . Evaluate the double integral

$$\int \int_S 4xy - y^3 \, dy \, dx.$$

(5 marks)

(ii) Let R be the region $\{(x, y) : 1 \leq y \leq 2, \ y \leq x \leq y^3\}$. Sketch the region denoted by R . Evaluate the double integral

$$\int \int_R \exp\left(\frac{x}{y}\right) \, dx \, dy.$$

(5 marks)

8 Find and classify the stationary points of the function

$$xy^2 - x^2 - 2y^2 = 0.$$

(10 marks)

End of Question Paper