



The  
University  
Of  
Sheffield.

**SCHOOL OF MATHEMATICS AND STATISTICS**

**Autumn Semester  
2017–18**

**Medical Statistics**

**2 hours**

*Candidates may bring to the examination a calculator that conforms to University regulations. All questions will be marked, but credit will be given for only the best **THREE** answers. All questions carry equal marks. Total marks 60.*

**Please leave this exam paper on your desk  
Do not remove it from the hall**

Registration number from U-Card (9 digits)  
to be completed by student

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**1** If babies are born prematurely, there is a risk that they will be under-developed and suffer a variety of health problems. There is good evidence that mothers who give birth prematurely in one pregnancy will also do so in later pregnancies, so in the following studies, the subjects were mothers who had given birth prematurely in their first pregnancy.

(i) Clinicians designed and conducted a study to assess whether a drug extended pregnancy, by comparing duration of the second pregnancy for two independent groups each of 40 mothers. One group was given the active drug and the other a placebo version. The results of the trial (in weeks) were as follows:

| Group   | mean | s.d. |
|---------|------|------|
| Drug    | 39.1 | 1.45 |
| Placebo | 38.2 | 1.60 |

(a) Verify that the trial had an adequate sample size to test for a difference of 1 week, when results expected on the placebo were a mean duration of 38 weeks, with a standard deviation of 1.5 weeks.

*(4 marks)*

(b) Did the trial provide evidence that the drug is beneficial in terms of extending pregnancy?

*(4 marks)*

(ii) When writing up their study, as well as assessing the effect on duration of pregnancy, the clinicians decide to examine whether the drug has any effect on the birthweights of the babies (hoping that longer durations will give the babies more time to develop and hence increase birthweights). Differences in mean birthweight between the two groups give a p-value for the t-test of  $p=0.03$ . What should you conclude about the effectiveness of the drug for increasing birthweights? Explain your reasoning. Does this change the way you would evaluate the effect of the drug on pregnancy duration in (i)(b)?

*(6 marks)*

(iii) In fact, it is subsequently revealed that one of the clinicians involved in the trial did not wait until the end of the trial for the formal test of effect of the drug on duration. Instead he looked at all the available data at 3 interim time points, after roughly each quarter of the patients had had their babies. A summary of the t-tests he conducted at these time points is:

| Group             | Time point 1 | Time point 2 | Time point 3 |
|-------------------|--------------|--------------|--------------|
| Number on Drug    | 10           | 20           | 30           |
| Number on Placebo | 9            | 20           | 31           |
| p-value           | 0.081        | 0.065        | 0.056        |

(a) Give 2 reasons why this might have been sensible.

*(2 marks)*

(b) Explain why there might be a problem in adopting this approach.

*(2 marks)*

(c) Was he right to let the trial run its full course?

*(2 marks)*

- 2 (i) A dermatologist investigating the use of emollient creams in managing dermatitis conducts a 6-week long crossover trial of a new cream versus a standard cream. Patients, who all suffered from a similar form of dermatitis of the hand, were allocated at random to either Group 1 or Group 2 and were instructed to apply the cream supplied (in an unbranded jar) to moisturize their hands twice daily. Group 1 used the new cream for 3 weeks, then reverted to the standard cream, while Group 2 continued with the standard cream for 3 weeks, then trialled the new cream for a further 3 weeks. Patients were encouraged to keep a diary of their impressions of the creams over the 6-week trial period (eg whether they experienced any flare ups or side effects, counting (approximately) the number of skin lesions, reporting the dryness of the skin), but ultimately only recorded whether they preferred their first or second cream (or whether they had no preference). The results are given below, coded as to whether the expressed preference was for new (denoted n), standard (denoted s) or neither (denoted –):

| Group          | patient | preference |
|----------------|---------|------------|
|                | 1       | n          |
|                | 2       | –          |
|                | 3       | n          |
|                | 4       | n          |
| Group 1        | 5       | n          |
| new → standard | 6       | –          |
|                | 7       | n          |
|                | 8       | n          |
|                | 9       | n          |
|                | 10      | n          |
|                | 11      | –          |
|                | 12      | n          |
|                | 13      | n          |
|                | 14      | –          |
| Group 2        | 15      | n          |
| standard → new | 16      | s          |
|                | 17      | s          |
|                | 18      | n          |
|                | 19      | s          |
|                | 20      | n          |

- (a) Explain why the creams were provided to patients in unbranded jars. *(2 marks)*
- (b) Comment on the form of response finally used (i.e. that patients kept a diary, but summarized their whole experience into a simple preference). *(2 marks)*
- (c) Show that the trial provides a clear indication that the new cream is preferable. *(5 marks)*
- (d) Show that there is also a suggestion of a period effect. *(3 marks)*

2 (continued)

- (ii) It has been suggested that future similar trials could be shortened to only 3 weeks, by applying the creams simultaneously; one to the patient's left hand and the other to their right hand.
  - (a) Explain why this could still be regarded as a form of crossover trial. *(2 marks)*
  - (b) Explain what key benefit (other than giving quicker results) this would offer over the format used in (i). *(2 marks)*
  - (c) What problems might arise if the patients did not all have the same dominant hand (i.e. some were right handed, others left handed)? *(2 marks)*
  - (d) What further modification of the trial format could you suggest to overcome the problem in (c)? Would it have any effect on your analysis? *(2 marks)*

3 A food manufacturer wants to test out a new diet program which they claim helps people lose weight more quickly. To analyse their claim they randomly allocate participants to either their new diet plan (plan A with 9 participants) or the current plan of a competitor (plan B with 8 participants). The table below shows the time taken until the participants lose 5kg in weight. An asterisk denotes a right censored observation where contact with the participant was lost before the participant lost the 5kg.

|       | New Plan A<br>Time (weeks) | Competitor Plan B<br>Time (weeks) |
|-------|----------------------------|-----------------------------------|
|       | 1                          | 2                                 |
|       | 2                          | 4                                 |
|       | 5*                         | 8                                 |
|       | 6                          | 9*                                |
|       | 7*                         | 12                                |
|       | 14                         | 15                                |
|       | 15*                        | 23*                               |
|       | 22                         | 23                                |
|       | 36                         |                                   |
| Total | 108                        | 96                                |

- (i) Estimate the value of the survivor function for the new diet plan at 25 weeks via Kaplan-Meier. *(5 marks)*
- (ii) Using your Kaplan-Meier estimate, write down an estimate for the median time for a participant on the new plan to lose 5kg. *(2 marks)*

**Note: You are not required to work out the Kaplan-Meier estimates for the competitor plan.**

3 (continued)

- (iii) It is suggested that the times to lose the weight in each group are exponentially distributed with rates  $\lambda_A$  and  $\lambda_B$  respectively. Under this assumption:
- (a) Estimate  $\lambda_A$  and  $\lambda_B$  and hence the mean time to lose 5kg with approximate 95% confidence intervals. *(4 marks)*
  - (b) How might you assess this assumption of exponentially distributed times? *(2 marks)*
- (iv) Using an exponential model as in (iii), perform a likelihood ratio test to assess whether there is a difference between the diet plans in terms of the time taken to lose 5kg. *(4 marks)*
- (v) The advertising authority is concerned about the possibility that the censoring may be informative. Explain what this means and give two possible reasons that could have led to informative censoring in this study. *(3 marks)*

- 4 (i) A new treatment is being trialled for lung cancer. 703 patients who presented with lung cancer were randomly allocated to either the new or standard treatment and followed up until either death or censoring. The information stored on each patient was:

|         |                                             |
|---------|---------------------------------------------|
| time:   | time until death/censoring (years)          |
| status: | failure indicator (1 = death; 0 = censored) |
| sex:    | Female or Male                              |
| drug:   | New or Standard                             |

The following R analysis was performed:

```
> lcancer.surv <- Surv(time, status)
>
> lcancer.fit <- coxph(lcancer.surv ~ sex + drug)
> summary(lcancer.fit)
Call:
coxph(formula = lcancer.surv ~ sex + drug)

n= 703, number of events= 703

              coef exp(coef) se(coef)      z Pr(>|z|)
sexMale      0.30019   1.35012  0.08017  3.744 0.000181 ***
drugStandard 0.42560   1.53052  0.07680  5.542  3e-08 ***
---
              exp(coef) exp(-coef) lower .95 upper .95
sexMale           1.350     0.7407    1.154    1.580
drugStandard      1.531     0.6534    1.317    1.779

Concordance= 0.556 (se = 0.012 )
Rsquare= 0.059 (max possible= 1 )
Likelihood ratio test= 43.04 on 2 df,  p=4.516e-10
Wald test              = 43.01 on 2 df,  p=4.58e-10
Score (logrank) test = 43.49 on 2 df,  p=3.606e-10
```

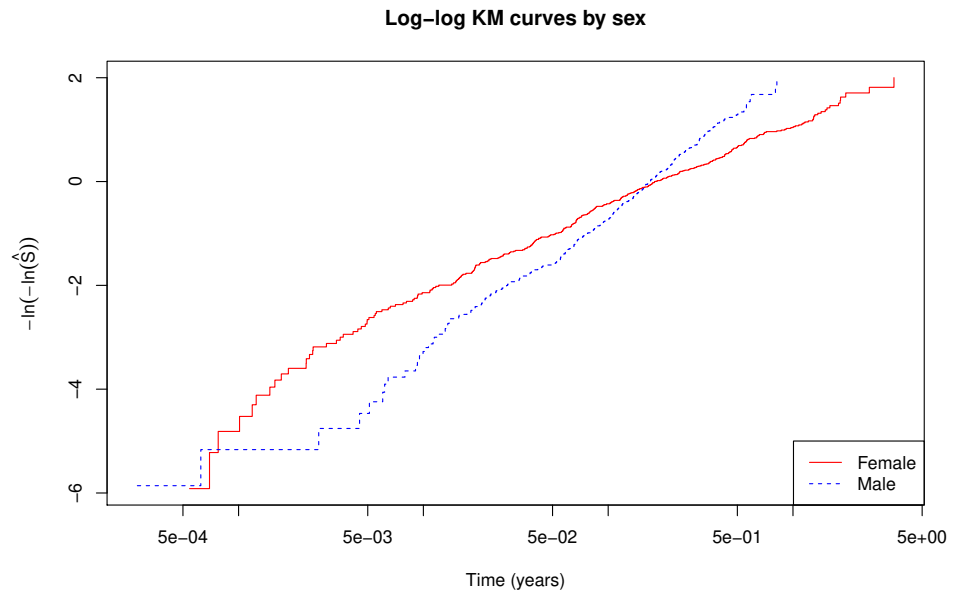
- (a) What type of model is fitted in `lcancer.fit`? Specify the form of the hazard for an individual given their sex and drug treatment. Make sure to describe your notation carefully. *(4 marks)*
- (b) Describe the findings of the analysis. *(4 marks)*

4 (continued)

(c) A statistician has performed some further analysis as shown below

```
> lcancer.fit2 <- coxph(lcancer.surv ~ drug + strata(sex))
> plot(survfit(lcancer.fit2), fun="cloglog")
```

which produced the following plot:



Does this plot raise any concerns regarding the suitability of the initial analysis? Justify your decision. *(3 marks)*

- (ii) A group of 126 elderly patients were studied to determine potential risk factors for hip fractures. The study investigators collected data on each patient's sex (male or female); age (measured in years); and whether they exercised (true or false).



4 (continued)

The patients were studied for a period of 5 years to see whether they fractured their hip in that time. Coding for the different variables is shown below:

|           |                                                                              |
|-----------|------------------------------------------------------------------------------|
| failtime: | time until hip fracture (years)                                              |
| status:   | fracture indicator (1 = hip fracture observed; 0 = no hip fracture observed) |
| sex:      | Female or Male                                                               |
| age:      | uncentred age of patient (years)                                             |
| exercise: | True or False                                                                |

An R analysis is provided below:

```
> hip.surv <- Surv(failtime, status)
>
> hipfrac.fit <- survreg(hip.surv ~ sex + age + exercise, dist = "exponential")
> summary(hipfrac.fit)
```

Call:

```
survreg(formula = hip.surv ~ sex + age + exercise, dist = "exponential")
```

|              | Value   | Std. Error | z     | p        |
|--------------|---------|------------|-------|----------|
| (Intercept)  | 3.8820  | 0.902      | 4.30  | 1.68e-05 |
| sexMale      | 0.6163  | 0.181      | 3.41  | 6.40e-04 |
| age          | -0.0279 | 0.010      | -2.78 | 5.51e-03 |
| exerciseTRUE | 0.3775  | 0.183      | 2.06  | 3.89e-02 |

Scale fixed at 1

Exponential distribution

Loglik(model)= -367.3 Loglik(intercept only)= -380.5

Chisq= 26.39 on 3 degrees of freedom, p= 7.9e-06

Number of Newton-Raphson Iterations: 5

n= 126

- (a) What type of model is fitted in `hipfrac.fit`? Write down the model fitted for the time  $T$  until hip fracture for an individual given their age, sex and exercise status. Make sure to describe your notation carefully. *(5 marks)*
- (b) Briefly describe the results of the analysis in terms of the effects of age, sex and exercise upon time to hip fracture. *(4 marks)*

**End of Question Paper**

# STANDARD FORMULAE FOR MEDICAL STATISTICS (INCLUDING TABLES OF CRITICAL VALUES)

## 1 Clinical Trials Formulae

**Two Sample t-Test — Separate variances form**  $r = \min(n_1, n_2)$

$$t_r = \left| \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \right|$$

**Two Sample t-Test — Pooled variance form**  $r = n_1 + n_2 - 2$

$$t_r = \left| \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2} \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}} \right|$$

**Sample Size Calculations — Two sample test for proportions** NB number in each group

$$n \simeq \frac{\theta_2(1-\theta_2) + \theta_1(1-\theta_1)}{(\theta_2 - \theta_1)^2} [\Phi^{-1}(\beta) + \Phi^{-1}(\alpha/2)]^2$$

**Sample Size Calculations — Two sample test for means** NB number in each group

$$n \simeq \frac{2\sigma^2}{(\mu_2 - \mu_1)^2} [\Phi^{-1}(\beta) + \Phi^{-1}(\alpha/2)]^2$$

**Standard Error for Natural Logarithm of Relative Risk**

$$s.e.[(\log_e(RR))] = \sqrt{\frac{1}{a} - \frac{1}{a+b} + \frac{1}{c} - \frac{1}{c+d}}$$

**Standard Error for Natural Logarithm of Odds Ratio**

$$s.e.[(\log_e(OR))] = \sqrt{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}}$$

## 2 Survival Analysis Formulae

**Exponential Distributions — MLE of rate  $\lambda$  with censoring** The mle

$$\hat{\lambda} = \frac{\sum_{i=1}^n \delta_i}{\sum_{i=1}^n t_i} = \frac{\Delta}{\mathcal{T}} \quad \text{var}(\hat{\lambda}) \approx \frac{\hat{\lambda}^2}{\sum_{i=1}^n \delta_i}.$$

For any (differentiable, monotonic) function  $g(\cdot)$ ,

$$\text{var}(g(\hat{\lambda})) \approx [\{g'(\lambda)\}^2 \text{var}(\lambda)]_{\lambda=\hat{\lambda}}.$$

so e.g.

$$\text{var}\left(\frac{1}{\hat{\lambda}}\right) = \text{var}(\hat{\mu}) \approx \frac{\hat{\mu}^2}{\sum_{i=1}^n \delta_i}$$

**Exponential Distributions — MLE test**

$$W = \frac{\hat{\lambda}_1 - \hat{\lambda}_2}{\sqrt{\frac{\hat{\lambda}_1^2}{\Delta_1} + \frac{\hat{\lambda}_2^2}{\Delta_2}}} \approx N(0, 1).$$

**Exponential Distributions — LRT test**

$$2 \left\{ \Delta_1 \log \frac{\Delta_1}{\mathcal{T}_1} + \Delta_2 \log \frac{\Delta_2}{\mathcal{T}_2} - (\Delta_1 + \Delta_2) \log \frac{\Delta_1 + \Delta_2}{\mathcal{T}_1 + \mathcal{T}_2} \right\} \approx \chi_1^2$$

**Log-rank Statistic**

$$LR = \frac{(O_1 - E_1)^2}{E_1} + \frac{(O_2 - E_2)^2}{E_2} \sim \chi_1^2$$

### 3 Tables of Percentage Points (also known as Quantiles or Critical Values) for Three Standard Distributions

The tables contain values of quantiles  $q$  such that  $P[X \leq q] = p$  for various probabilities  $p$  when  $X$  has the specified distribution (which may depend on particular degrees of freedom  $\nu$ ). In these tables,  $p$  has been expressed as a percentage rather than a decimal. The relevant  $R$  commands for generating the  $q$  are also shown. For the  $N(0, 1)$  distribution, the tabulated function is also known as the  $\Phi^{-1}$  function.

#### STANDARD NORMAL DISTRIBUTION PERCENTAGE POINTS

`qnorm(p)` where  $p$  is percentage, e.g. for 95%,  $p = 0.95$

|                    | 60.0% | 66.7% | 75.0% | 80.0% | 87.5% | 90.0% | 95.0% | 97.5% | 99.0% | 99.5% | 99.9% |
|--------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| <code>qnorm</code> | 0.253 | 0.431 | 0.674 | 0.842 | 1.150 | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | 3.090 |

#### CHI-SQUARED PERCENTAGE POINTS

`qchisq(p, nu)` where  $p$  is percentage, e.g. for 95%,  $p = 0.95$

| $\nu$ | 60.0%  | 66.7%  | 75.0%  | 80.0%  | 87.5%  | 90.0%  | 95.0%  | 97.5%  | 99.0%  | 99.5%  | 99.9%  |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 1     | 0.708  | 0.936  | 1.323  | 1.642  | 2.354  | 2.706  | 3.841  | 5.024  | 6.635  | 7.879  | 10.828 |
| 2     | 1.833  | 2.197  | 2.773  | 3.219  | 4.159  | 4.605  | 5.991  | 7.378  | 9.210  | 10.597 | 13.816 |
| 3     | 2.946  | 3.405  | 4.108  | 4.642  | 5.739  | 6.251  | 7.815  | 9.348  | 11.345 | 12.838 | 16.266 |
| 4     | 4.045  | 4.579  | 5.385  | 5.989  | 7.214  | 7.779  | 9.488  | 11.143 | 13.277 | 14.860 | 18.467 |
| 5     | 5.132  | 5.730  | 6.626  | 7.289  | 8.625  | 9.236  | 11.070 | 12.833 | 15.086 | 16.750 | 20.515 |
| 6     | 6.211  | 6.867  | 7.841  | 8.558  | 9.992  | 10.645 | 12.592 | 14.449 | 16.812 | 18.548 | 22.458 |
| 7     | 7.283  | 7.992  | 9.037  | 9.803  | 11.326 | 12.017 | 14.067 | 16.013 | 18.475 | 20.278 | 24.322 |
| 8     | 8.351  | 9.107  | 10.219 | 11.030 | 12.636 | 13.362 | 15.507 | 17.535 | 20.090 | 21.955 | 26.125 |
| 9     | 9.414  | 10.215 | 11.389 | 12.242 | 13.926 | 14.684 | 16.919 | 19.023 | 21.666 | 23.589 | 27.877 |
| 10    | 10.473 | 11.317 | 12.549 | 13.442 | 15.198 | 15.987 | 18.307 | 20.483 | 23.209 | 25.188 | 29.588 |

STUDENT'S  $t$  PERCENTAGE POINTS  
 $qt(p, \nu)$  where  $p$  is percentage, e.g. for 95%,  $p = 0.95$

| $\nu$    | 60.0% | 66.7% | 75.0% | 80.0% | 87.5% | 90.0% | 95.0% | 97.5%  | 99.0%  | 99.5%  | 99.9%  |
|----------|-------|-------|-------|-------|-------|-------|-------|--------|--------|--------|--------|
| 1        | 0.325 | 0.577 | 1.000 | 1.376 | 2.414 | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 | 318.31 |
| 2        | 0.289 | 0.500 | 0.816 | 1.061 | 1.604 | 1.886 | 2.920 | 4.303  | 6.965  | 9.925  | 22.327 |
| 3        | 0.277 | 0.476 | 0.765 | 0.978 | 1.423 | 1.638 | 2.353 | 3.182  | 4.541  | 5.841  | 10.215 |
| 4        | 0.271 | 0.464 | 0.741 | 0.941 | 1.344 | 1.533 | 2.132 | 2.776  | 3.747  | 4.604  | 7.173  |
| 5        | 0.267 | 0.457 | 0.727 | 0.920 | 1.301 | 1.476 | 2.015 | 2.571  | 3.365  | 4.032  | 5.893  |
| 6        | 0.265 | 0.453 | 0.718 | 0.906 | 1.273 | 1.440 | 1.943 | 2.447  | 3.143  | 3.707  | 5.208  |
| 7        | 0.263 | 0.449 | 0.711 | 0.896 | 1.254 | 1.415 | 1.895 | 2.365  | 2.998  | 3.499  | 4.785  |
| 8        | 0.262 | 0.447 | 0.706 | 0.889 | 1.240 | 1.397 | 1.860 | 2.306  | 2.896  | 3.355  | 4.501  |
| 9        | 0.261 | 0.445 | 0.703 | 0.883 | 1.230 | 1.383 | 1.833 | 2.262  | 2.821  | 3.250  | 4.297  |
| 10       | 0.260 | 0.444 | 0.700 | 0.879 | 1.221 | 1.372 | 1.812 | 2.228  | 2.764  | 3.169  | 4.144  |
| 11       | 0.260 | 0.443 | 0.697 | 0.876 | 1.214 | 1.363 | 1.796 | 2.201  | 2.718  | 3.106  | 4.025  |
| 12       | 0.259 | 0.442 | 0.695 | 0.873 | 1.209 | 1.356 | 1.782 | 2.179  | 2.681  | 3.055  | 3.930  |
| 13       | 0.259 | 0.441 | 0.694 | 0.870 | 1.204 | 1.350 | 1.771 | 2.160  | 2.650  | 3.012  | 3.852  |
| 14       | 0.258 | 0.440 | 0.692 | 0.868 | 1.200 | 1.345 | 1.761 | 2.145  | 2.624  | 2.977  | 3.787  |
| 15       | 0.258 | 0.439 | 0.691 | 0.866 | 1.197 | 1.341 | 1.753 | 2.131  | 2.602  | 2.947  | 3.733  |
| 16       | 0.258 | 0.439 | 0.690 | 0.865 | 1.194 | 1.337 | 1.746 | 2.120  | 2.583  | 2.921  | 3.686  |
| 17       | 0.257 | 0.438 | 0.689 | 0.863 | 1.191 | 1.333 | 1.740 | 2.110  | 2.567  | 2.898  | 3.646  |
| 18       | 0.257 | 0.438 | 0.688 | 0.862 | 1.189 | 1.330 | 1.734 | 2.101  | 2.552  | 2.878  | 3.610  |
| 19       | 0.257 | 0.438 | 0.688 | 0.861 | 1.187 | 1.328 | 1.729 | 2.093  | 2.539  | 2.861  | 3.579  |
| 20       | 0.257 | 0.437 | 0.687 | 0.860 | 1.185 | 1.325 | 1.725 | 2.086  | 2.528  | 2.845  | 3.552  |
| 21       | 0.257 | 0.437 | 0.686 | 0.859 | 1.183 | 1.323 | 1.721 | 2.080  | 2.518  | 2.831  | 3.527  |
| 22       | 0.256 | 0.437 | 0.686 | 0.858 | 1.182 | 1.321 | 1.717 | 2.074  | 2.508  | 2.819  | 3.505  |
| 23       | 0.256 | 0.436 | 0.685 | 0.858 | 1.180 | 1.319 | 1.714 | 2.069  | 2.500  | 2.807  | 3.485  |
| 24       | 0.256 | 0.436 | 0.685 | 0.857 | 1.179 | 1.318 | 1.711 | 2.064  | 2.492  | 2.797  | 3.467  |
| 25       | 0.256 | 0.436 | 0.684 | 0.856 | 1.178 | 1.316 | 1.708 | 2.060  | 2.485  | 2.787  | 3.450  |
| 26       | 0.256 | 0.436 | 0.684 | 0.856 | 1.177 | 1.315 | 1.706 | 2.056  | 2.479  | 2.779  | 3.435  |
| 27       | 0.256 | 0.435 | 0.684 | 0.855 | 1.176 | 1.314 | 1.703 | 2.052  | 2.473  | 2.771  | 3.421  |
| 28       | 0.256 | 0.435 | 0.683 | 0.855 | 1.175 | 1.313 | 1.701 | 2.048  | 2.467  | 2.763  | 3.408  |
| 29       | 0.256 | 0.435 | 0.683 | 0.854 | 1.174 | 1.311 | 1.699 | 2.045  | 2.462  | 2.756  | 3.396  |
| 30       | 0.256 | 0.435 | 0.683 | 0.854 | 1.173 | 1.310 | 1.697 | 2.042  | 2.457  | 2.750  | 3.385  |
| 35       | 0.255 | 0.434 | 0.682 | 0.852 | 1.170 | 1.306 | 1.690 | 2.030  | 2.438  | 2.724  | 3.340  |
| 40       | 0.255 | 0.434 | 0.681 | 0.851 | 1.167 | 1.303 | 1.684 | 2.021  | 2.423  | 2.704  | 3.307  |
| 45       | 0.255 | 0.434 | 0.680 | 0.850 | 1.165 | 1.301 | 1.679 | 2.014  | 2.412  | 2.690  | 3.281  |
| 50       | 0.255 | 0.433 | 0.679 | 0.849 | 1.164 | 1.299 | 1.676 | 2.009  | 2.403  | 2.678  | 3.261  |
| 55       | 0.255 | 0.433 | 0.679 | 0.848 | 1.163 | 1.297 | 1.673 | 2.004  | 2.396  | 2.668  | 3.245  |
| 60       | 0.254 | 0.433 | 0.679 | 0.848 | 1.162 | 1.296 | 1.671 | 2.000  | 2.390  | 2.660  | 3.232  |
| $\infty$ | 0.253 | 0.431 | 0.674 | 0.842 | 1.150 | 1.282 | 1.645 | 1.960  | 2.326  | 2.576  | 3.090  |