



The
University
Of
Sheffield.

MAS004

SCHOOL OF MATHEMATICS AND STATISTICS

**Spring Semester
2018–2019**

Further Foundation Mathematics

2 hours

Answer all questions. Total marks 80.

You should justify your answers carefully unless the question states otherwise.

- 1 (i) Let $z = 1 + i$.
- (a) Indicate z , z^2 and z^{-1} all on the same Argand diagram. *(3 marks)*
 - (b) Using your answer to part (a), find $(1 + i)^{16}$. *(3 marks)*
 - (c) Express $i^{1/2}$ in the form $x + iy$. *(2 marks)*
- (ii) Find the expressions for and sketch the loci of the following
- (a) $|z - 2i| = |z - 2|$.
 - (b) $|z - 3 - 2i| = 2$.
 - (c) $|z - 3 - 2i| = |z - 1 - i|$. *(12 marks)*

2 In this question, let $\mathbf{u} = \mathbf{i} + 3\mathbf{j} + \mathbf{k}$, $\mathbf{v} = 2\mathbf{j} - 2\mathbf{k}$, and $\mathbf{w} = 3\mathbf{i} + 5\mathbf{j} + 5\mathbf{k}$.

(i) (a) Evaluate $\mathbf{u} \times \mathbf{v}$, $\mathbf{u} \cdot \mathbf{w}$ and $\mathbf{v} \cdot \mathbf{w}$. *(4 marks)*

(b) Hence evaluate $(\mathbf{u} \times \mathbf{v}) \times \mathbf{w}$ and $\mathbf{v}(\mathbf{u} \cdot \mathbf{w}) - \mathbf{u}(\mathbf{v} \cdot \mathbf{w})$. *(4 marks)*

(ii) Consider three particles with position vectors

$$\mathbf{a} = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} t, \quad \mathbf{b} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} + \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix} t,$$

and

$$\mathbf{c} = \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ -3 \\ 3 \end{pmatrix} t.$$

Find if any of the particle paths cross or collide, and state the time of the collision, if one is found. *(12 marks)*

3 (i) Sketch a graph of the function

$$f(x) = \frac{x^3 + 3x^2 + x - 5}{x(x - 1)},$$

marking clearly any points of interest, i.e., turning points, discontinuities and axis crossings. *(6 marks)*

(ii) (a) Evaluate $\prod_{p=1}^5 (2^p - 1)$. *(3 marks)*

(b) If $m = 7$, evaluate $\frac{1}{3} \sum_{n=3}^m (m - 2n)$. *(3 marks)*

(iii) Showing your working carefully, find the Maclaurin series with terms of degree up to and including x^4 of $f(x) = e^x x^2$. *(8 marks)*

- 4 (i) (a) Find, by the method of separation of variables or otherwise, the general solution to the differential equation

$$x^{-2} \frac{dy}{dx} = 1 - y.$$

(7 marks)

- (b) Hence find a solution which has $y = 2$ when $x = 0$. *(3 marks)*

- (ii) (a) Find the general solution to the differential equation

$$2 \frac{d^2y}{dx^2} - 4 \frac{dy}{dx} - \frac{5}{2}y = 0.$$

(6 marks)

- (b) Hence find a solution which has $y = 1$ and $\frac{dy}{dx} = 0$ when $x = 0$.

(4 marks)

End of Question Paper