The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **Spring Semester 2018–2019**

History of Mathematics **2 hours 30 minutes**

Answer Question 1 and three other questions. If you answer more than three of the Questions 2 to 5 only your best three will be counted.

1 Attempt *three* of questions **(a)**, **(b)**, **(c)**, **(d)** below. If you attempt *more* than three, only your best three will be counted.

(a) Give an account of the *area of a circle* in Egyptian mathematics. ***(7 marks)***

(b) State the *first term* and the *common difference* of the arithmetic progression, the sum of whose first n terms is the n th m-gonal number. Deduce that the sum of the first n *odd* numbers is n^2, and that the n th m-gonal number is $\left(\frac{m-2}{2}\right)n^2 - \left(\frac{m-4}{2}\right)n$. ***(7 marks)***

(c) Give the context of the *first* appearance on the mathematical stage of the following problem: *A tree 12 units high is broken in two at a point where the height of the part remaining is the cube root of the length of the part cut away. What is the height of the part remaining?* Give the context of its *first* appearance *in print*. [You are *not* expected to solve the problem.] ***(7 marks)***

(d) State Archimedes' *formula* for the area of a parabolic segment. Give *two* works in which he established it, naming the method of proof employed in each case. Use his *formula* to show that $\int_0^a x^2\,dx = \frac{1}{3}a^3\ (a > 0)$. ***(7 marks)***

2 **(a)** Name the clay tablet on which the sexagesimal number $1\,;\,24, 51, 10$ is inscribed. Write the number in (i) decimal form correct to *six* decimal places; (ii) in cuneiform, as it appears on the tablet itself. What is its significance? ***(5 marks)***

(b) Which clay tablet (studied in the course) contains the following problem: *I add seven times the side of my square and eleven times the area*: $6\,;15$. What is this problem? Find its positive solution using conventional methods, writing it in Babylonian cuneiform. What evidence suggests that this tablet was used in a teaching context? ***(5 marks)***

(c) Write the first *three* numbers on a row of *Plimpton 322*, the *eleventh* excepted, in terms of two *regular sexagesimals* p and q $(p > q)$. On the fifth row, the *second* and *third* numbers are 65 and 97, find the *first* to four decimal places. ***(6 marks)***

3 Outline the structure of **Book I** of Euclid's *Elements*, giving illustrative examples. ***(10 marks)***

What do you consider its weaknesses as: **(a)** a learned mathematical treatise**;** **(b)** a student textbook**;** **(c)** a source book for the historian of mathematics? ***(6 marks)***

4 The following titles are of books by *three* British mathematicians. *A Briefe and True Report on the New Found Land of Virginia*; *A Plaine Discovery of the Whole Revelation of Saint John***;** *The Urinal of Physick.* For each book, name its author and indicate its contents. Place the works in chronological order. ***(5 marks)***

Which *two* of the authors you named above, published *mathematics* books in their lifetimes? For each, give the title of one such book, indicate what motivated its publication, and comment on its success. ***(6 marks)***

Which author published *no* mathematics book during his lifetime? Why might this have been? What has become of his research papers since his death? ***(5 marks)***

5 **(a)** State the three-dimensional form of Cavalieri's Principle. ***(2 marks)***

In three dimensions, a circular disc of radius r, centre $(R, 0, 0)$ $(R > r)$, lies in the (x, z)-plane, and is rotated about the z-axis to form a solid *torus* T. A *cylinder* C with axis the y-axis, radius r, altitude $2\pi R$, has centre the origin.

For each h $(-r < h < r)$, denote by T_h (respectively C_h) the intersection of T (respectively C) with the plane $z = h$. Given that T_h is an annulus with inner radius $R-\sqrt{r^2 - h^2}$ and outer radius $R+\sqrt{r^2 - h^2}$, and that C_h is a rectangle with sides $2\sqrt{r^2 - h^2}$ and $2\pi R$. Show that T_h and C_h have *equal* areas. Deduce that T has volume $2\pi^2 r^2 R$. ***(7 marks)***

(b) How might Fermat have shown that the area under the curve $y = 1/\sqrt{x}$ and above the interval $(0, a]$ on the *x-axis*, where $a > 0$, is $2\sqrt{a}$? ***(7 marks)***

End of Question Paper