



The
University
Of
Sheffield.

MAS5050

SCHOOL OF MATHEMATICS AND STATISTICS

**Spring Semester
2018–2019**

Mathematical Methods for Statistics

2 hours

RESTRICTED OPEN BOOK EXAMINATION

Candidates may bring to the examination lecture notes and associated lecture material (including set textbooks) plus a calculator that conforms to University regulations.

*Candidates should attempt **ALL** questions.*

The paper will be marked out of 80 and the allocation of marks is shown in brackets.

**Please leave this exam paper on your desk
Do not remove it from the hall**

Registration number from U-Card (9 digits)
to be completed by student

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- 1** (i) The sum of the first n terms of a series is $n^2 + 3n$.
- (a) Find the n th term of the series. **(3 marks)**
- (b) Show that the series is arithmetic and find the first term and the common difference. **(2 marks)**
- (ii) Using appropriate formulae for the sums of series, evaluate $\sum_{n=1}^7 16 \left(-\frac{1}{2}\right)^n$ and find the sum to infinity of the series. **(5 marks)**

- 2** Use Gaussian elimination to solve the following system of equations:

$$\begin{aligned} x + 5y - 2z &= -2 \\ 4x + 10y - 6z &= 12 \\ 5x + 20y + z &= 50 \end{aligned}$$

(10 marks)

- 3** Compute the derivatives of the following functions with respect to x :

(i) $r(x) = \left(\frac{a-x}{a+x}\right)^{\frac{1}{2}}$, where a is a constant. **(5 marks)**

(ii) $s(x) = \ln\left(\frac{b-x}{b+x}\right)$, where b is a constant. **(3 marks)**

(iii) $t(x) = \sin(\cos x)$. **(2 marks)**

- 4** (i) By making a suitable substitution determine the indefinite integral

$$\int 5x(2x^2 + 1)^{-3} dx.$$

(4 marks)

- (ii) By making suitable substitutions compute the definite integral

$$\int_0^{\frac{\pi}{3}} \left(3 \sin\left(\frac{z}{2}\right) - 5 \cos(\pi - z)\right) dz.$$

(6 marks)

5 (i) Let

$$A = \begin{pmatrix} 2 & 2 \\ 5 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 6 & 3 \\ 1 & 2 \end{pmatrix}.$$

(a) Calculate $AB + BA$. (2 marks)

(b) Find A^{-1} . (2 marks)

(ii) Find the eigenvalues of A and their associated eigenvectors. (6 marks)

6 (i) Let R be the region $\left\{ (x, y) : 0 \leq x \leq 2, 0 \leq y \leq \frac{x}{2} \right\}$.
 Sketch the region denoted by R .
 Evaluate the double integral

$$\int \int_R xy^2 \, dy \, dx.$$

(4 marks)

(ii) Let S be the region bounded by the lines $y = x$, $x + y = \frac{\pi}{2}$ and $y = 0$.
 Sketch the region denoted by S . Evaluate the double integral

$$\int \int_S \sin(x + y) \, dx \, dy.$$

(6 marks)

7 (i) Find, to the nearest degree, the angle θ between the vectors $\mathbf{p} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$
 and $\mathbf{q} = 3\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}$. (5 marks)

(ii) Find a unit vector perpendicular to the plane that contains the vectors
 $2\mathbf{i} - \mathbf{j} + \mathbf{k}$ and $3\mathbf{i} + 4\mathbf{j} - \mathbf{k}$. (5 marks)

8 Find and classify the stationary points of the function

$$f(x, y) = \frac{x^3}{3} - x + \frac{y^2}{2} + 2y.$$

(10 marks)

End of Question Paper