The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **Spring Semester 2019–2020**

Quantum Theory

This is an open book exam.
Answer ***all*** *questions.*
The submission deadline is 10 am (BST), twenty-four hours after it is released. Late submission will not be considered without extenuating circumstances. It is expected that you will be able to complete this exam in approximately one hour and it is recommended that you submit the work within four hours. You will not be penalised for taking longer, however.
Unless it is explicitly stated otherwise, it is intended that calculations are performed by hand (possibly with the aid of a calculator). To gain full marks, you will need to show your working. You will not get full marks if you simply write down output from a computer package.
By uploading your solutions you declare that your submission consists entirely of your own work, that any use of sources or tools other than material provided for this module is cited and acknowledged and that no unfair means have been used.

Please leave this exam paper on your desk
Do not remove it from the hall

Registration number from U-Card (9 digits)
to be completed by student

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1 (i) Let $|1\rangle$ be the state

$$|1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad .$$

Furthermore, let the Hamiltonian of a quantum system have the form

$$\hat{H} = \begin{pmatrix} h & g \\ g & h \end{pmatrix} \quad ,$$

where h and g are real numbers.

- Give a reason why it is important in quantum mechanics that the Hamiltonian operator is in general self-adjoint? Give a **short** answer. ***(6 marks)***

- Find the eigenvalues and normalised eigenvectors of $\hat{H}$. ***(16 marks)***

- If the system starts out at $t = 0$ in the state $|1\rangle$, what is its state at a later time t? ***(14 marks)***

(ii) Let

$$A = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

be an operator. Let the states $|\psi_0\rangle$ and $|\psi_1\rangle$ be defined as

$$|\psi_0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

and

$$|\psi_1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Show that

$$A\left(\frac{1}{\sqrt{2}}|\psi_0\rangle - \frac{1}{\sqrt{2}}|\psi_1\rangle\right) = |\psi_1\rangle$$

(14 marks)

2 (i) Consider the three operators

$$\begin{aligned}\mathcal{O}_1 &= -i\hbar x\frac{\partial}{\partial x}\\ \mathcal{O}_2 &= \hbar x\frac{\partial}{\partial x}\\ \mathcal{O}_3 &= -i\hbar\left(x\frac{\partial}{\partial x}+\frac{\partial}{\partial x}x\right)\end{aligned}$$

For each of these operators, determine whether they are linear and self-adjoint. Assume that wave functions $\Psi(x,t)$ are normalisable and that $x\Psi(x,t) \to 0$ for $x \to \pm\infty$. ***(40 marks)***

(ii) Consider the operator

$$\hat{H} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V_1(x) - iV_2(x),$$

where $V_1(x)$ and $V_2(x)$ are real functions of x. Is it linear? Is this operator self-adjoint? Give a **short** answer. ***(10 marks)***

End of Question Paper