The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **Spring Semester 2019–2020**

Knots and Surfaces

This is an open book exam.
*Answer **all** questions.*
The submission deadline is 10 am (BST), twenty-four hours after it is released. Late submission will not be considered without extenuating circumstances. It is expected that you will be able to complete this exam in approximately one hour and it is recommended that you submit the work within four hours. You will not be penalised for taking longer, however.
Unless it is explicitly stated otherwise, it is intended that calculations are performed by hand (possibly with the aid of a calculator). To gain full marks, you will need to show your working. You will not get full marks if you simply write down output from a computer package.
By uploading your solutions you declare that your submission consists entirely of your own work, that any use of sources or tools other than material provided for this module is cited and acknowledged and that no unfair means have been used.

Please leave this exam paper on your desk
Do not remove it from the hall

Registration number from U-Card (9 digits)
to be completed by student

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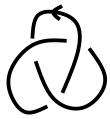
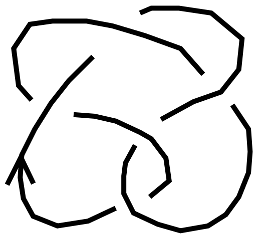
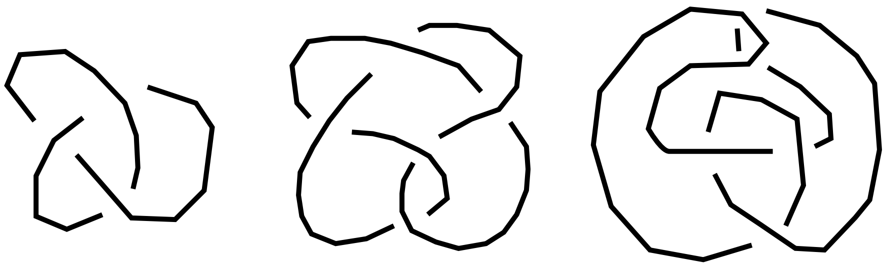
1 (i) In this question you may use any property about the Jones polynomial given in the lecture notes provided that it is clearly stated. You may also use the fact that the Jones polynomials of the right-handed trefoil knot  is given by

V$(A) = -A^{-16} + A^{-12} + A^{-4}$.

(a) Calculate the Jones polynomial of the following knot **(10 marks)**



(b) Determine which of the following knots are equivalent and which are distinct. Your answer should include a rigorous justification. **(5 marks)**



1 (continued)

(ii) Throughout this question you may assume that the C-polynomial

$$C : \{\text{oriented link diagrams}\} \longrightarrow \mathbb{Z}[x]$$
$$D \longmapsto C_D(x)$$

is the well-defined oriented link invariant determined by:

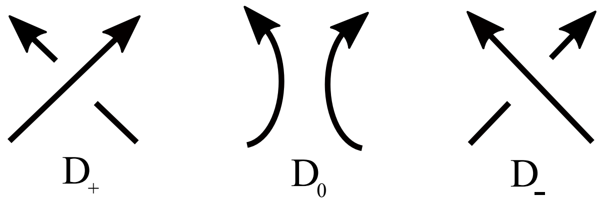
(normalisation) $C_U(x) = 1$ for the unknot U;

(invariance) if D and D' are Reidemeister equivalent then $C_D(x) = C_{D'}(x)$;

(skein relation)

$$C_{D_+}(x) - C_{D_-}(x) \ = \ xC_{D_0}(x)$$

where D_+, D_- and D_0 are link diagrams which differ only at a single crossing as shown below



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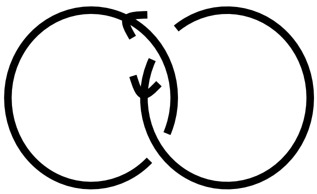
(a) Show that $C_{D \sqcup U}(x) = 0$ for every link diagram D, where $D \sqcup U$ represents the link diagram D together with a disjoint copy of the unknot.

(5 marks)

Hint: Consider a diagram containing a crossing of the form .

(b) Calculate $C_H(x)$ where H is the oriented Hopf link shown below.

(5 marks)



2 (i) Determine whether the following statements are true or false. Your answer should include a proof or counterexample to justify your answer.

(a) If two subsets A and B of $\mathbb{R}^3$ are homeomorphic then they are ambient isotopic. **(5 marks)**

(b) If $f, g : \mathbb{R}^2 \to \mathbb{R}$ are continuous functions then the graphs of f and g are homeomorphic. i.e.

$$\{(x, y, f(x, y)) \mid (x, y) \in \mathbb{R}^2\} \;\cong\; \{(x, y, g(x, y)) \mid (x, y) \in \mathbb{R}^2\}.$$

(5 marks)

(ii) Let ω be the surface word given by

$$\omega \;=\; xxa^{-1}bcyyb^{-1}c^{-1}a.$$

(a) Determine the standard form of the surface $S(\omega)$ by calculating the surface's Euler characteristic. **(5 marks)**

(b) Determine the standard form of the surface $S(\omega)$ by exhibiting an explicit sequence of word operations. **(5 marks)**

(c) Determine with proof, whether $S(\omega)$ is homeomorphic to $S(\omega')$ where ω' is the surface word given by

$$\omega' \;=\; abcf^{-1}defa^{-1}b^{-1}c^{-1}e^{-1}d^{-1}.$$

(5 marks)

End of Question Paper