The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **Spring Semester 2019–2020**

Time Series **1 hour**

This is an open book exam.
*Answer **all** questions. The allocation of marks is shown in brackets.*
There are 50 marks available on the paper.

The submission deadline is 10 am (BST), twenty-four hours after it is released. Late submission will not be considered without extenuating circumstances. It is expected that you will be able to complete this exam in approximately one hour and it is recommended that you submit the work within four hours. You will not be penalised for taking longer, however.

Unless it is explicitly stated otherwise, it is intended that calculations are performed by hand (possibly with the aid of a calculator). To gain full marks, you will need to show your working. You will not get full marks if you simply write down output from a computer package.

By uploading your solutions you declare that your submission consists entirely of your own work, that any use of sources or tools other than material provided for this module is cited and acknowledged and that no unfair means have been used.

Please leave this exam paper on your desk
Do not remove it from the hall

Registration number from U-Card (9 digits)
to be completed by student

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1 (i) A time series $\{y_t\}$ consisting of 8 observations on quarterly average of daily temperatures (in degrees Celsius) over two years is shown in the table below.

	Year 1				Year 2			
Quarter	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
t	1	2	3	4	5	6	7	8
y_t	8	13	16	10	6	11	15	12

In order to analyse this data the decomposition method is considered, according to which the time series y_t is decomposed into trend m_t, seasonal s_t and residual components r_t, so that

$$y_t = m_t + s_t + r_t.$$

(a) Using a 3-point moving average calculate estimates of m_t, for $t = 2, 3, \ldots, 7$. ***(2 marks)***

(b) Provide estimates of the seasonals s_{Q1}, s_{Q2}, s_{Q3} and s_{Q4}, for quarters Q1, Q2, Q3 and Q4. ***(6 marks)***

(c) Calculate estimates of the residual term r_t, for $t = 2, 3, \ldots, 7$. ***(5 marks)***

(ii) Consider the time series y_t is generated from the time series model

$$y_t = \epsilon_t + \frac{1}{2}\epsilon_{t-1} - \frac{1}{4}\epsilon_{t-2},$$

where ϵ_t is a white noise sequence with variance 2.

(a) Show that y_t is invertible. ***(2 marks)***

(b) Calculate the variance of y_t. ***(1 mark)***

(c) Provide the autocorrelation function (ACF) of y_t. ***(5 marks)***

(d) Calculate the first two values $a_1^{(1)}$ and $a_2^{(2)}$ of the partial autocorrelation function (PACF) of y_t. ***(4 marks)***

2 The price p_t of an asset traded in the stock market follows the evolution given below

$$p_t = p_{t-1}\exp(r_t),$$

where r_t denotes the logarithmic return of the asset at time t.

A time series model for the returns r_t is adopted given by

$$r_t = 0.8r_{t-1} + \kappa_t,$$

where κ_t is a white noise process, following a normal distribution with variance 4.

Define the state vector

$$\beta_t = \begin{bmatrix} \log p_{t-1} \\ r_t \end{bmatrix}.$$

(i) Find a state-space model for the log-price $\log p_t$ of A, i.e.

$$\log p_t = x_t^T\beta_t + \epsilon_t$$
$$\beta_t = F\beta_{t-1} + \zeta_t$$

and determine x_t and F and state the distributions of ϵ_t and ζ_t.

(8 marks)

(ii) After 100 trading days the posterior distribution of β_{100}, given data $p_{1:100} = \{p_1, p_2, \ldots, p_{100}\}$ is:

$$\beta_{100} \mid p_{1:100} \sim N\left\{\begin{bmatrix} 5 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}\right\}.$$

(a) Provide the 2-step ahead forecast distribution of $\log p_{102}$, given data $p_1, p_2, \ldots, p_{100}$. ***(15 marks)***

(b) Obtain a 95% predictive interval for the return r_{102}, given data $p_1, p_2, \ldots, p_{100}$. ***(2 marks)***

End of Question Paper