The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **Spring semester 2019-2020**

Operations Research **1 Hour**

This is an open book exam.
*Answer **all** questions.*
The submission deadline is 10 am (BST), twenty-four hours after it is released. Late submission will not be considered without extenuating circumstances. It is expected that you will be able to complete this exam in approximately one hour and it is recommended that you submit the work within four hours. You will not be penalised for taking longer, however.
Unless it is explicitly stated otherwise, it is intended that calculations are performed by hand (possibly with the aid of a calculator). To gain full marks, you will need to show your working. You will not get full marks if you simply write down output from a computer package.
By uploading your solutions you declare that your submission consists entirely of your own work, that any use of sources or tools other than material provided for this module is cited and acknowledged and that no unfair means have been used.

1 (i) Teleco plans to produce four types of telecommunication devices, temporarily code named P1, P2, P3 and P4. The table below provides data on the cost of parts, machine time required and the profit estimates for each device.

	P1	P2	P3	P4
Cost of parts per unit (£)	5	3	8	6
Machine time per unit (hrs)	4	3	5	4
Profit per unit (£)	3	2	2	4

It is also known that:

- The available capital for parts is estimated at £30000.
- 4600 hours machine time is available without cost. A one-off additional 200 hours machine time can be made available at a cost of £2000.
- There is a one-time setup cost if a device is made. The setup cost for making P1 is £500. P2 and P3 are made on the same assembly line that costs £300 to set up. The setup cost for P4 is negligible.
- P1 and P4 have similar functionalities. To avoid self-competition, the difference between the numbers of P1 and P4 should be no less than 50% of the total number of the two.

Formulate the mixed integer linear programming model for Teleco to maximise the total profit. **Find the formulation only; do NOT attempt to find the numerical solution**. ***(16 marks)***

1 (continued)

(ii) A manufacturer is considering making four types of cars: C1, C2, C3, and C4. The following table gives both the assembly and testing times for each car along with the profit.

	C1	C2	C3	C4	Availability (days)
Assembly time (days/unit)	1	3	8	4	90
Testing time (days/unit)	1	1	1	3	80
Profit (k£/unit)	1	2	4	3	

The company needs to determine the optimal production schedule that maximises the total profit. Letting x_1, x_2, x_3 and x_4 be the numbers of C1, C2, C3 and C4 to be made, and x_5 and x_6 be the slack variables corresponding to the assembly and testing time constraints, respectively, the linear programming model for the optimal schedule is formulated as follows:

$$\begin{aligned} \max \quad & z = x_1 + 2x_2 + 4x_3 + 3x_4\,, \\ \text{subject to} \quad & x_1 + 3x_2 + 8x_3 + 4x_4 \leq 90\,, \\ & x_1 + x_2 + x_3 + 3x_4 \leq 80\,. \end{aligned}$$

As a first approximation, the variables are treated as non-integer variables. Using the simplex method, the optimal tableau is found to be:

	x_1	x_2	x_3	x_4	x_5	x_6	Solution
z	0	0	$1/2$	$1/2$	$1/2$	$1/2$	85
x_2	0	1	$7/2$	$1/2$	$1/2$	$-1/2$	5
x_1	1	0	$-5/2$	$5/2$	$-1/2$	$3/2$	75

Now suppose technology upgrades have made it possible to make one C3 car with only 4 days of assembly time and 2 days of testing. Does the solution remain optimal? Give your reasons. If it does not remain optimal, find the new optimal tableau. ***(9 marks)***

End of Question Paper