MAS110 Exam 2020-21

Instructions

- This is an open book exam.
- Answer all questions.
- You can work on the exam during the 24 hour period starting at 10am (GMT) ***put in exam date***, and you must submit your work within 2 hours and 30 minutes of accessing the exam paper or by the end of the 24 hour period (whichever is earlier). Late submission will not be considered without extenuating circumstances.
- Unless it is explicitly stated otherwise, it is intended that calculations are performed by hand (possibly with the aid of a calculator).
- By completing the assessment you declare that your submission consists entirely of your own work, that any use of sources or tools other than material provided for this module is cited and acknowledged, and that no unfair means have been used.

Further instructions.

- There are 17 questions in this test. All questions are compulsory. The allocation of marks is shown next to the question.
- Questions 1 to 12 are multiple choice questions. Each question has four options, and you need to select the unique correct option.
- Questions 14, 15 and 16 are numerical calculations. The result, or results, are all integers.
- Questions 16 and 17 are multiple answer questions. There are six options, and you need to select the correct option or options. You will get partial credit for each correct choice. However, any incorrect choice will result in an overall score of 0 marks for the question.

1. If $A = \{1, 2, 3, 4, 5, 6\}$, then the cardinality of the set

$$\{\,(a, b, c) \in A^3 \mid a \text{ is even, } b \text{ is odd}\,\}$$

is

A. $3!3!$ **B.** 3×3 **C.** $3 \times 3 \times 3$ **D.** $3 \times 3 \times 6$

2. $\dfrac{\sin(A+B) - \sin(A-B)}{\cos(A-B) - \cos(A+B)} =$

A. $\tan A$ **B.** $\cot A$ **C.** $\tan B$ **D.** $\cot B$

3. What is the argument of the complex number $-1 - \sqrt{3}i$?

A. $\dfrac{4\pi}{3}$ **B.** $\dfrac{\pi}{3}$ **C.** $\tan^{-1}(1/\sqrt{3})$ **D.** $\tan^{-1}(-\sqrt{3})$

4. $(1+i)^{2020} =$

A. 2^{1010} **B.** -2^{1010} **C.** $i2^{1010}$ **D.** $-i2^{1010}$

5. The function

$$f(x) = \begin{cases} x^2 + 1, & \text{for } x \geq 3 \\ 2x + k, & \text{for } x < 3 \end{cases}$$

is continuous at $x = 3$. Then $k =$

A. 3 **B.** 4 **C.** 5 **D.** $\dfrac{5}{2}$

6. If $\lim_{x \to 0} \dfrac{f(x)}{x} = 5$, then $\lim_{x \to 0} \dfrac{f(3x)}{\sin(x)} =$

A. 5 **B.** 3 **C.** 15 **D.** no unique answer

7. The differentiable functions $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ satisfy $f(0) = 1$, $f'(0) = 2$, $g(0) = 3$ and. $g'(0) = 4$. Then the slope of the tangent to the curve $y = f(x)g(x)$ at $x = 0$ is

A. 0 **B.** 10 **C.** 14 **D.** 11

8. If $f : \mathbb{R} \to \mathbb{R}$ is differentiable at 1 with derivative 1, then $\lim_{h\to 0} \dfrac{f(1-h^2) - f(1)}{h^2} =$

A. 0 **B.** 1 **C.** -1 **D.** not enough data

9. If $\int_0^8 f(x)\,dx = 6$, then $\int_0^2 x^2 f(x^3)\,dx =$

A. 0 **B.** 1 **C.** 2 **D.** $\dfrac{4}{3}$

10. The radius of convergence of the power series $x^2 - x^3 + x^4 - x^5 + \dots$ is

A. 0 **B.** 1 **C.** 2 **D.** -1

11. The continuous function $f : \mathbb{R} \to \mathbb{R}$ satisfies

$$\int f(x)\cos(x)\,dx = f(x)\sin(x) - \int 4x^3 \sin(x)\,dx.$$

Then $f(x)$ could be

A. $\cos(x)$ **B.** x^4 **C.** $-x^4$ **D.** $4x^3$

12. $\lim_{n\to\infty} \dfrac{1^{2020} + 2^{2020} + \cdots + n^{2020}}{n^{2021}} =$

A. 0 **B.** 2021 **C.** $\dfrac{1}{2020}$ **D.** $\dfrac{1}{2021}$

13. The differentiable function $f : \mathbb{R} \to \mathbb{R}$ satisfies $f(1) = 2$ and $f'(1) = \frac{1}{\ln(2)}$. Evaluate

$$\frac{d}{dx}\left((1+x)^{f(x)}\right)$$

at $x = 1$.

14. If $F(x) = \int_4^{x^2} \sqrt{1 + t^{3/2}}\,dt$, then $\lim_{x\to 2} \dfrac{F(x)}{x^2 - 4} =$

15. Given real numbers $a, b, c \in \mathbb{R}$ such that

$$\lim_{x\to 0} \frac{\cos(x^2) - a + bx^4}{cx^8} = 1,$$

calculate the following quantities.

A. a **B.** $\frac{b}{c}$ **C.** $\frac{ab}{c}$

16. Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be two functions. Given that the composite $f \circ g : \mathbb{R} \to \mathbb{R}$ is bijective, which of the following statements must definitely be true?

(A) f is injective.
(B) f is surjective.
(C) f is bijective.
(D) g is injective.
(E) g is surjective.
(F) g is bijective.

17. Let $f : \mathbb{R} \to \mathbb{R}$ be the function given by

$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & \text{when } x > 0, \\ x^2, & \text{when } x \leq 0. \end{cases}$$

Which of the following statements are correct?

(A) f is **not** continuous at 0.
(B) f is continuous everywhere.
(C) f is **not** differentiable at 0.
(D) f is differentiable everywhere and $f'(0) = \lim_{x\to 0^+} f'(x)$.
(E) f is differentiable everywhere and $f'(0) = \lim_{x\to 0^-} f'(x)$.
(F) The derivative of f is a differentiable function on the whole of $\mathbb{R}$.