The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **Autumn Semester 2020–21**

Game Theory

This is an open book exam.
Answer ***all*** *questions.*
You can work on the exam during the 24 hour period starting at 10am (GMT), and you must submit your work within 3 hours of accessing the exam paper or by the end of the 24 hour period (whichever is earlier). ***Late submission will not be considered without extenuating circumstances.*** *Unless it is explicitly stated otherwise, it is intended that calculations are performed by hand (possibly with the aid of a calculator). To gain full marks, you will need to show your working. By uploading your solutions you declare that your submission consists entirely of your own work, that any use of sources or tools other than material provided for this module is cited and acknowledged, and that no unfair means have been used.*

1 Four people are given the choice of a quiet holiday in different locations A, B and C.

People choose simultaneously and independently a location, and obtain the following utilities.

- A person who chose a location chosen by more than one person gets zero utility.
- A person who is the only person to have chosen A gets 1 unit of utility.
- A person who is the only person to have chosen B gets 1 unit of utility.
- A person who is the only person to have chosen C gets 8 units of utility.

(i) Describe all pure Nash equilibria of this setup. ***(4 marks)***

(ii) Describe a mixed-strategy Nash equilibrium in which each person plays the same mixed strategy. ***(6 marks)***

2 Alice and Bob invest in two competing projects. They both can choose to invest any non-negative amount of money. If Alice invests x million pounds and Bob invests y million pounds, their profits will be $A(x, y) = xy^2 - x^2$ and $B(x, y) = xy - y^2 + 2y$, respectively, in units of millions of pounds.

(i) Find the best response functions for Alice and Bob. ***(6 marks)***

(ii) Find all Nash equilibria of this setup. ***(4 marks)***

3 Alice and Bob face the following game

	l	r
U	3, 8	6, 6
D	1, 2	8, 3

and choose to negotiate an outcome, with the knowledge that, if they fail to strike a deal, utilities of 3 will be imposed on both.

Answer the following multiple choice question (select *one* answer).

The payoffs of the Nash Bargain of this setup are

(a) $9/2$ for Alice and 7 for Bob

(b) 6 for Alice and 6 for Bob

(c) 7 for Alice and $9/2$ for Bob

(d) None of the above

(8 marks)

4 Alice and Bob play the following game. A fixed matrix $n \times n$ matrix M is given, and let $X \subset \mathbb{R}$ be the set of entries in M. Alice and Bob alternate making moves, with Alice going first. When it is Alice's turn she deletes a row of M which has not been previously deleted. When it is Bob's turn he deletes a column of M which has not been previously deleted. The game ends when there are one row r and one column c left, and then the payoff for Alice is M_{rc} and the payoff for Bob is $-M_{rc}$.

(i) Prove that for any subset S of X, either Alice has a strategy which guarantees her a payoff in S, or Bob has a strategy that guarantees him a payoff of $-s$ with s in X but not in S. State any theorems used in your proof. ***(6 marks)***

(ii) Let

$$M = \begin{bmatrix} 6 & 0 \\ 3 & 2 \end{bmatrix}.$$

Describe the game corresponding to this M in normal form, and find all Nash equilibria of the game. ***(6 marks)***

5 Alice and Bob play repeatedly the following game given in strategic form as follows:

	I	II
X	64, 64	20, 72
Y	72, 20	57, 57

Answer the following multiple choice questions.

(i) If this game is repeated 13 times, what is the total payoff for Alice at a Nash equilibrium of this repeated game? (Select *one* answer).

(a) 260

(b) 640

(c) 741

(d) 936

(4 marks)

(ii) This game is repeated indefinitely, with each repetition occurring with probability $p < 1$. What is the largest range of values of p for which the proof of the corresponding Folk Theorem guarantees an expected payoff of $64/(1-p)$ for both players? (Select *one* answer).

(a) $57/72 < p < 1$

(b) $57/64 < p < 1$

(c) $8/15 < p < 1$

(d) None of the above

(6 marks)

End of Question Paper