The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **Spring 2020–21**

FOUNDATION YEAR CORE MATHEMATICS

2.5 hours

This is an open book exam.

Answer all questions.

You can work on the exam during the 24 hour period starting from 10am (BST), and you must submit your work within 2.5 hours of accessing the exam paper or by the end of the 24 hour period (whichever is earlier).

Late submission will not be considered without extenuating circumstances.
Calculations should be performed by hand. A university-approved calculator may be used, but it is intended that calculations are performed by hand.
The use of any other calculational device, software or service is not permitted. To gain full marks, you will need to show your working.

By uploading your solutions you declare that your submission consists entirely of your own work, that any use of sources or tools other than material provided for this module is cited and acknowledged, and that no unfair means have been used.

Answers to this paper must be written on clean sides of paper and show clearly your working.
Total marks: 42.

Please leave this exam paper on your desk
Do not remove it from the hall

Registration number from U-Card (9 digits)
to be completed by student

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1 Solve the equation below for β.

$$4\log_k 2 - \log_k 5 - \log_k \beta = \log_k \beta - \log_k 10.$$

(4 marks)

2 (i) By using the definition of the binomial coefficient, show that

$$\binom{m}{n} = \binom{m}{m-n}$$

for $m > n$ where $m \in \mathbb{N}$ and $n \in \mathbb{N} \cup \{0\}$. ***(3 marks)***

(ii) Evaluate $\dfrac{12!}{9!}$ efficiently. ***(1 mark)***

3 By using the compound angle formulae for sine and cosine, derive the compound angle formula for $\cot(\theta + \varphi)$, involving *only* $\cot\theta$ and $\cot\varphi$ as the trigonometric functions in the resulting expression.

Explain briefly each step of your calculation. ***(4 marks)***

4 The probability distribution of a discrete random variable, V, is as follows:

v	1	2	3
P_v	0.15	0.30	P_3

where P_v denotes the probability that $V = v$.

Find $\mathrm{Var}(V)$, simplifying your answer as much as possible. ***(5 marks)***

5 Recall the properties that:

- **(Prop. 1):** the scalar product distributes, i.e. $\boldsymbol{p} \bullet (\boldsymbol{q} + \boldsymbol{r} + \boldsymbol{s}) = \boldsymbol{p} \bullet \boldsymbol{q} + \boldsymbol{p} \bullet \boldsymbol{r} + \boldsymbol{p} \bullet \boldsymbol{s}$;
- **(Prop. 2):** $\boldsymbol{p} \bullet \boldsymbol{q} = 0$ iff $\boldsymbol{p}$ is perpendicular to $\boldsymbol{q}$;
- **(Prop. 3):** and that $\boldsymbol{p} \bullet \boldsymbol{p} = |p|^2$ for any vector $\boldsymbol{p}$.

Let $\boldsymbol{v} = \boldsymbol{i} - 2\boldsymbol{j} - 3\boldsymbol{k}$ and $\boldsymbol{w} = 3\boldsymbol{i} - 2\boldsymbol{j} - \boldsymbol{k}$.

Show that $\boldsymbol{v} \bullet \boldsymbol{w} = 10$, justifying your solution by applying each of the properties above to the Cartesian unit basis vectors. ***(4 marks)***

6 Let $y = f(x) = \dfrac{x^2}{x^3 - 1}$.

(i) Find the stationary points of $f(x)$ and determine their nature. ***(7 marks)***

(ii) Sketch the graph of $y = f(x)$, stating the domain of the function and clearly marking or stating the x- and y-intercepts, stationary points, behaviour where the function is not defined and the behaviour of $f(x)$ as $x \to \pm\infty$, if applicable.

(6 marks)

(iii) Show that the *area* A enclosed by $y = \dfrac{x^2}{x^3 - 1}$, the x-axis, the y-axis and $x = -e$ is $A = \dfrac{1}{3}\ln(e^3 + 1)$, giving clear reasons for your answer. ***(4 marks)***

7 Find

$$\int e^{2x} \cos(3x + 1)\, dx.$$

(4 marks)

End of Question Paper