The University Of Sheffield.

**SCHOOL OF MATHEMATICS AND STATISTICS** **Spring Semester 2020–2021**

**Further Foundation Mathematics** **2 hours**

*This is an open book exam.*
*Answer all three questions. The total marks for this exam is 60.*

*You can work on the exam during the 24 hour period starting at 10am (BST), and you must submit your work within 2 hours of accessing the exam paper or by the end of the 24 hour period (whichever is earlier).* ***Late submission will not be considered without extenuating circumstances.*** *Calculations should be performed by hand. A university-approved calculator may be used. The use of any other calculational device, software or service is not permitted. To gain full marks, you will need to show your working.*

*By uploading your solutions you declare that your submission consists entirely of your own work, that any use of sources or tools other than material provided for this module is cited and acknowledged, and that no unfair means have been used.*

**Please leave this exam paper on your desk**
**Do not remove it from the hall**

Registration number from U-Card (9 digits)
to be completed by student

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**1** (i) Let $u = 3 + 2i$, $v = 2 + i$, and $z = -2 + 2i$.

Find the modulus and the argument for each of $u, v$, and $z$.
Give the arguments in radians, as an exact value if possible (e.g., if a multiple of $\pi$) or to four decimal places if not. ***(5 marks)***

(ii) (a) Let $C$ be the locus of points satisfying the equation

$$x^2 + 12x + y^2 - 8y + 27 = 0.$$

Give a geometric description of $C$. ***(4 marks)***

(b) Find the coordinates of the points where $C$ meets the straight line which has equation $y = 2 - 2x$. ***(3 marks)***

(iii) For this question, let $\mathbf{u} = 3\mathbf{i} - 2\mathbf{j} - 5\mathbf{k}$, $\mathbf{v} = \mathbf{i} - 2\mathbf{k}$ and $\mathbf{w} = \mathbf{j} + \mathbf{k}$.

(a) What is the angle between $\mathbf{u}$ and $\mathbf{w}$?

(b) What is the angle between $\mathbf{u}$ and $-\mathbf{w}$?

Give your answer in radians to three decimal places. ***(5 marks)***

(iv) Consider the plane of points whose position vectors $\mathbf{a}$ satisfy the equation

$$\mathbf{a} = \mathbf{u} + \lambda\mathbf{v} + \mu\mathbf{w},$$

for real numbers $\lambda$ and $\mu$. The vectors $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ are as in part (iii) above.

(a) Which of the vectors $3\mathbf{i}$, $3\mathbf{j}$ and $3\mathbf{k}$ lie on this plane? ***(6 marks)***

(b) Write down a vector perpendicular to this plane. ***(4 marks)***

**2** (i) Determine, with justification, whether the following function is continuous:

$$f(x) = \begin{cases} \dfrac{1}{2} - 2x^2 + \dfrac{5}{2}x^3, & \text{if } \ x \leq 1, \\ \ln(|x-2|), & \text{if } \ 1 < x \leq \dfrac{7}{4}. \end{cases}$$

***(3 marks)***

(ii) Find the first four terms of the Maclaurin series for $e^{2x}$.

***(5 marks)***

(iii) (a) Write down the terms up to (and including) the $x^3$ term of the binomial series for $\dfrac{1}{(1-3x)^2}$. ***(5 marks)***

(b) Multiply this series by $1 - 6x + 9x^2$ by hand, and comment on what you find. ***(3 marks)***

**3** (i) (a) Show that $y = x\ln(x)$ is a solution to the differential equation

$$x\frac{dy}{dx} = x + y.$$

***(3 marks)***

(b) Let $a$ be an arbitrary constant. Show that $y = a\tan(ax)$ is a solution to the differential equation

$$\frac{dy}{dx} = a^2 + y^2.$$

***(6 marks)***

(ii) (a) Find the general solution to the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 15y = 0.$$

***(5 marks)***

(b) Hence find a solution to this equation which has $y = 1$ and $\frac{dy}{dx} = 1$ when $x = 0$. ***(3 marks)***

## End of Question Paper