



The
University
Of
Sheffield.

MAS112

SCHOOL OF MATHEMATICS AND STATISTICS

**Spring Semester
2020–2021**

Vectors and Mechanics

2.5 hours

This is an open book exam.

Answer all questions.

*You can work on the exam during the 24 hour period starting at 10am (GMT), and you must submit your work within 2.5 hours of accessing the exam paper or by the end of the 24 hour period (whichever is earlier). **Late submission will not be considered without extenuating circumstances.** Calculations should be performed by hand. A university-approved calculator may be used. The use of any other calculational device, software or service is not permitted. To gain full marks, you will need to show your working. By uploading your solutions you declare that your submission consists entirely of your own work, that any use of sources or tools other than material provided for this module is cited and acknowledged, and that no unfair means have been used. **Note that all vectors must be clearly underlined.** Use the value of $g = 9.8 \text{ ms}^{-2}$ when needed.*

Total marks 40

- 1 Let ABC be a triangle. Let D be the point on BC such that $BD/DC = 1/2$, let E be the point on AC such that $AE/EC = 1/3$ and let F be the mid-point of AD .
 - (i) Using position vectors, show that B , F and E are collinear.
 - (ii) Find the value of BF/FE . **(8 marks)**

- 2 Let $w = 3u + 5v$, where u and v denote unit vectors. The angle between u and v is $\pi/3$. Find $|w|$. **(3 marks)**

- 3 A line is defined by the vector equation $\mathbf{r} \times (-\mathbf{i} + \mathbf{j} + 3\mathbf{k}) = 3\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}$ and a plane is given by $\mathbf{r} \cdot (3\mathbf{i} - 2\mathbf{j} + \mathbf{k}) = 10$.
- (i) Find where the line and plane intersect.
- (ii) Find the acute angle in radians between the line and the plane to an accuracy of two decimal places. *(9 marks)*
- 4 A force $\mathbf{F} = 5\mathbf{i} + 3\mathbf{j}$ is applied on an object at a point P with coordinates $4\mathbf{i}$, which then moves it to point B with coordinates $3\mathbf{j}$. Calculate the work done by the force. *(4 marks)*
- 5 What is the magnitude of the acceleration of a small spot of Blu Tack on the edge of a turntable turning at 6 radians/sec if the turntable's diameter is 0.3 m? *(3 marks)*
- 6 What is the maximum speed with which a 1500 kg lorry can round a turn of radius 98.5 m on a flat road if the coefficient of friction between the lorry tyre and the flat road is 0.7? Round your answer to two significant figures. *(2 marks)*
- 7 A bacterium in a fluid is subjected to a viscous drag force $\mathbf{F} = -b\mathbf{v}$, where, with respect to a Cartesian co-ordinate system, $\mathbf{v} = v(t)\mathbf{i}$ is the velocity of the bacterium relative to the fluid. The parameter b is a positive constant and the minus sign is due to the drag force being opposite to the direction of motion. The bacterium moves with an initial velocity relative to the fluid of $v_0\mathbf{i}$. Use the following ordinary differential equation derived from Newton's second law

$$m \frac{dv}{dt} = -bv,$$

where m is the mass of the bacterium, to find an expression for $v(t)$. Given that at time $t = 0$, the bacterium is at $x = 0$, find an expression for the bacterium's position $x(t)$. Find the asymptote of $x(t)$ as $t \rightarrow \infty$. *(7 marks)*

- 8 A particle moves on a straight line through a fixed point O so that at time t its displacement from O is x . The equation of motion of the particle is

$$\ddot{x} + \omega^2 x = 0,$$

where ω is a positive constant. If, at time $t = 0$, we have $x = x_0$ and $\dot{x} = u$, find $x(t)$ in terms of x_0 , ω and u . *(4 marks)*

End of Question Paper