The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **2020–2021**

Analysis **1 hour 45 minutes**

This is an open book exam.

Answer all four questions.

You can work on the exam during the 24 hour period starting from 10am (BST), and you must submit your work within 1 hour and 45 minutes of accessing the exam paper or by the end of the 24 hour period (whichever is earlier).

Late submission will not be considered without extenuating circumstances. *Calculations should be performed by hand. A university-approved calculator may be used. The use of any other calculational device, software or service is not permitted. To gain full marks, you will need to show your working.*

By uploading your solutions you declare that your submission consists entirely of your own work, that any use of sources or tools other than material provided for this module is cited and acknowledged, and that no unfair means have been used.

1 Consider the sequence of real numbers (a_n) where $a_1 = 0$ and $a_n = 4 + \dfrac{1}{(n-1)n}$ for $n > 1$. What is the limit of this sequence? Give a careful proof that the sequence converges to this limit, using the ε, N definition. **(8 marks)**

2 Let $a, b \in \mathbb{R}$ with $a < b$ and let $f : [a, b] \to \mathbb{R}$ be a continuous function such that the image of f is $[-b, -a]$. Use the Intermediate Value Theorem to show that there is some $c \in [a, b]$ such that $f(c) = -c$. **(9 marks)**

3 Let $f : [0,4] \to \mathbb{R}$ be a monotonic increasing Riemann integrable function, such that $f(i) = i$ for $i \in \{0,1,2,3,4\}$. Show that

$$6 \leq \int_0^4 f(t)\,dt \leq 10,$$

justifying your answer carefully by direct reference to the relevant definitions for the Riemann integral. **(8 marks)**

4 Show that there is a function $f\colon [0,2\pi] \to \mathbb{R}$ defined by

$$f(x) = \sum_{n=1}^{\infty} \frac{\cos(nx)}{n(n+3)}$$

and that this function is continuous.

[You may assume that cos is a continuous function and that the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges.] **(10 marks)**

End of Question Paper