The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **Spring Semester 2020–2021**

Mathematics (Computational and Numerical Methods) **1.5 hours**

This is an open book exam.

Answer all questions.

You can work on the exam during the 24 hour period starting from 10am (BST), and you must submit your work within 1.5 hours of accessing the exam paper or by the end of the 24 hour period (whichever is earlier).

Late submission will not be considered without extenuating circumstances. *Calculations should be performed by hand. A university-approved calculator may be used. The use of any other calculational device, software or service is not permitted. To gain full marks, you will need to show your working.*

By uploading your solutions you declare that your submission consists entirely of your own work, that any use of sources or tools other than material provided for this module is cited and acknowledged, and that no unfair means have been used.

Total marks 30

1 To an accuracy of 4 decimal places the function $f(x) = \ln(x)$ produces the following $(x, f(x))$ data points: $(5, 1.6094)$, $(6, 1.7918)$ and $(7, 1.9459)$. If these data points are used for a quadratic interpolation, find the magnitudes of the upper and lower error bounds for the interpolated value at $x = 6.5$. Give these error bounds to an accuracy of 4 decimal places. ***(7 marks)***

2 A function $y(x)$ satisfies the ordinary differential equation

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + (5x+2)y = 0$$

and the conditions $y(0) = 1$ and $y'(0) = 0$. Use the Euler method with a step size of $h = 0.1$ to determine $y(0.3)$. Give your final answer to an accuracy of 3 decimal places.

Hint: Introduce the new variable $z = \dfrac{dy}{dx}$ and rewrite the second order ordinary differential equation as two coupled first order ordinary differential equations.

(8 marks)

3 A manufacturer makes products A and B. To produce each of these products two production processes, P_1 and P_2, are required. Product A requires 1 hour of P_1 and 3 hours of P_2, while product B requires 4 hours of P_1 and 2 hours of P_2. Each week the total time for production process P_1 is 25 hours and for P_2 it is 20 hours. Sales figures show that at least 1 unit of product B is sold for every 2 units of product A. The net profit per unit of product A is £ 600 and for a unit of product B it is £ 200.

Formulate this into a linear programming problem and use graphical methods to determine the maximum possible weekly profit. On the graph, clearly show the feasibility region and the line of constant revenue through the point of maximum weekly profit.

(15 marks)

End of Question Paper