The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **Spring semester 2020-2021**

Operations Research **2.5 hours**

This is an open book exam.

Answer all questions.

You can work on the exam during the 24 hour period starting from 10am (BST), and you must submit your work within 2.5 hours of accessing the exam paper or by the end of the 24 hour period (whichever is earlier).

Late submission will not be considered without extenuating circumstances. *Calculations should be performed by hand. A university-approved calculator may be used. The use of any other calculational device, software or service is not permitted. To gain full marks, you will need to show your working.*

By uploading your solutions you declare that your submission consists entirely of your own work, that any use of sources or tools other than material provided for this module is cited and acknowledged, and that no unfair means have been used.

1 A manufacturer makes two types of mining drills, called D1 and D2. The drills can be assembled with two different approaches. In the first approach, the drills are assembled using assembly line A alone. In the second approach, the drills go through assembly lines B and C in succession. The hours needed to assemble a drill on each assembly line, the available hours of each assembly line, and the unit profit of each drill are given below:

	A (hours)	B (hours)	C (hours)	Profit (k£/unit)
D1	15	5	7	14
D2	17	4	9	25
Weekly Availability	140	158	148	

The workforce hours needed to assemble a drill using the two approaches are different, and are given in the table below:

	Using line A	Using lines B and C
D1	3 hours	2 hours
D2	8 hours	6 hours

Additionally, the following information is known:

- The manufacturer will choose only one of the two approaches to assemble the drills.
- Using assembly lines B and C incurs additional £20k operational cost per week.
- The manufacturer needs to source a key component M1 from outside providers. D1 and D2 use 5 units and 7 units of M1, respectively.
- M1 can be purchased from a regular provider at a cost of £3k per unit for the first 20 units, and £2k per unit afterwards.
- The provider can supply at most 38 units of M1 each week. Additional units have to be purchased from another provider at a cost of £5k per unit.
- The manufacturer has 380 hours of free workforce available weekly. It has £550k capital available for purchasing M1.

Set up a mixed integer-linear programming model for the problem so that the manufacturer can use it to find the optimal production schedule to maximise its weekly profit. **Find the formulation only; do NOT try to find the numerical solution.**

(50 marks)

2 A paint factory produces three types of paints, called P1, P2, and P3, by mixing two raw materials A and B with a liquid solvent. The cost of the solvent is negligible and its amount is unlimited. The table below shows the amounts of A and B needed to mix one tonne of each paint, the availability of the raw materials, and the unit profit of each paint.

	P1	P2	P3	Availability (kg)
A(kg)	4	10	6	5300
B(kg)	6	8	12	5400
Unit profit (k£/tonne)	6	12	10	

To determine the production schedule that maximises the total profit, we define x_1, x_2 and x_3 as the amounts (in tonnes) of P1, P2, and P3 to be produced, respectively. The optimal schedule can be solved from the following linear programming problem:

$$\max \quad z = 6x_1 + 12x_2 + 10x_3 \qquad (\text{in k£})$$

subject to $x_1, x_2, x_3 \geq 0$, and

$$4x_1 + 10x_2 + 6x_3 \leq 5300,$$
$$6x_1 + 8x_2 + 12x_3 \leq 5400.$$

After introducing slack variables x_4 and x_5 for the first and the second constraints, respectively, the optimal tableau can be found with the standard primal simplex algorithm, which is given as follows:

Basis	x_1	x_2	x_3	x_4	x_5	Solution
z	0	0	2/7	6/7	3/7	48000/7
x_2	0	1	-3/7	3/14	-1/7	2550/7
x_1	1	0	18/7	-2/7	5/14	2900/7

(i) Find the optimal cost and the optimal solution for the primal variables from the optimal tableau. ***(2 marks)***

(ii) Write down, without proof, the dual linear programming problem. What is the optimal value for the dual cost function? What is the optimal solution for the dual variables? ***(6 marks)***

(iii) Find the optimality range for the cost coefficient of x_1. ***(6 marks)***

(iv) A technological upgrade would make it possible to produce 1 tonne of P3 with 4kg of material A and 8kg of B. Implementing the upgrade would incur a cost of £75k. Meanwhile, due to temporary shortage in storage space, the total amount of P2 and P3 to be produced has to be limited to no more than 400 tonnes. Under this additional constraint, is it worth it to implement the technological upgrade? You need to use calculation to support your answer; a simple Yes or No answer would not be given marks. ***(20 marks)***

 Question 2 continued on next page

2 (continued)

(v) Suppose the total amount of P2 and P3 is limited to no more than b_3 tonnes, what is the smallest b_3 for which the technological upgrade is still profitable? ***(6 marks)***

(vi) Suppose b_3 has to be reduced to 300. Find the new optimal solution. ***(10 marks)***

End of Question Paper