The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **Spring Semester 2020–2021**

Knots and Surfaces **3 hours**

This is an open book exam.

Answer all questions.

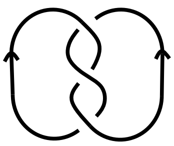
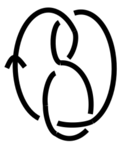
You can work on the exam during the 24 hour period starting from 10am (BST), and you must submit your work within 3 hours of accessing the exam paper or by the end of the 24 hour period (whichever is earlier).

Late submission will not be considered without extenuating circumstances. *Calculations should be performed by hand. A university-approved calculator may be used. The use of any other calculational device, software or service is not permitted. To gain full marks, you will need to show your working.*

By uploading your solutions you declare that your submission consists entirely of your own work, that any use of sources or tools other than material provided for this module is cited and acknowledged, and that no unfair means have been used.

1 (i) Let K denote the knot diagram below.

(a) Calculate the Jones polynomial of the knot K. You may use the table of Jones polynomials below in your calculation.

D	$V_D(A)$
	$-A^{-10} - A^{-2}$
	$-A^{16} + A^{12} + A^{4}$
	$A^{8} - A^{4} + 1 - A^{-4} + A^{-8}$

(10 marks)

(b) Is the knot K from Part (a) equivalent to its mirror image? You should fully justify your answer. **(5 marks)**

 Question 1 continued on next page

1 (continued)

(ii) A *nice 3-colouring* assigns one of {blue, green, red} to each arc of a link diagram so that at each crossing either all arcs are the same colour or all arcs are distinct colours. The colour counting function

$$N : \{\text{link diagrams}\} \to \mathbb{N}$$

counts the number of nice 3-colourings on a diagram and is a link invariant. Note that colouring all arcs on a diagram the same colour produces a nice 3-colouring, $N(\text{unknot}) = 3$, and $N(\text{right-handed trefoil}) = 9$.

(a) Verify that N is invariant under RII moves. **(5 marks)**

(b) Do there exist knot diagrams D_1, D_2 with $N(D_1), N(D_2) \neq 3$ and $N(D_1) \neq N(D_2)$? You should fully justify your answer. **(5 marks)**

2 (i) (a) Let

$$A = abca^{-1}b^{-1}c^{-1} \quad \text{and} \quad B = xyx^{-1}y^{-1}.$$

Show that A and B are word equivalent by exhibiting an explicit sequence of word moves. **(5 marks)**

(b) Let K denote the Klein bottle and T denote the torus. Determine the standard form of the surface $T\#K\#K\#K$. **(5 marks)**

(ii) Determine whether the following statements are true or false. Your answer should be fully justified with a rigorous proof or counterexample.

(a) If I is a bounded open interval of the real line and $B \subset \mathbb{R}^2$ is homeomorphic to I then B is bounded. **(5 marks)**

(b) If w_1 and w_2 are surface words with $\chi(S(w_1)) = \chi(S(w_2))$ then $S(w_1) \cong S(w_2)$. **(5 marks)**

(c) There does not exist a covering pattern of the projective plane which consists of triangles with 6 triangles meeting at each vertex. **(5 marks)**

End of Question Paper