



The  
University  
Of  
Sheffield.

**MAS5050**

**SCHOOL OF MATHEMATICS AND STATISTICS**

**Spring Semester  
2020–2021**

**Mathematical Methods for Statistics**

**2 hours**

*RESTRICTED OPEN BOOK EXAMINATION*

*Candidates may bring to the examination lecture notes and associated lecture material (including set textbooks) plus a calculator that conforms to University regulations.*

*Candidates should attempt **ALL** questions.*

*The paper will be marked out of 80 and the allocation of marks is shown in brackets.*

**Please leave this exam paper on your desk  
Do not remove it from the hall**

Registration number from U-Card (9 digits)  
to be completed by student

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**Blank**

- 1** Fill in the multiple blanks:  
The function  $f(x, y)$  is given by

$$f(x, y) = x^3 + x^2 - xy + y^2 + 5.$$

The number of stationary points that  $f(x, y)$  contains is [Blank1].  
Correct to two decimal places, the value of  $f(x, y)$  at its saddle point is [Blank2].  
**(8 marks)**

- 2** Fill in the multiple blanks:  
Use Gaussian elimination to solve the following system of equations:

$$\begin{aligned} x - 10y + z &= -12 \\ 4x + y - 6z &= 65 \\ -3x + 20y + 5z &= 14 \end{aligned}$$

The solution is given by  $x$ =[Blank1],  $y$ =[Blank2],  $z$ =[Blank3] **(9 marks)**

- 3** Fill in the multiple blanks:  
The vectors  $\mathbf{a}$ ,  $\mathbf{b}$  and  $\mathbf{c}$  are given by  $\mathbf{a} = (p, 0, -3)$ ,  $\mathbf{b} = (2, q, 4)$  and  $\mathbf{c} = (3, 5, r)$ , where  $p$ ,  $q$ , and  $r$  are unknown constants. Given that  $\mathbf{a} + 3\mathbf{b} = 3\mathbf{c}$ , find the following values:  $p$ =[Blank1],  $q$ =[Blank2] and  $r$ =[Blank3].  
**(3 marks)**

- 4** The vectors  $\mathbf{a} = 3\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$  and  $\mathbf{b} = t\mathbf{i} + 4\mathbf{j} - \mathbf{k}$ , where  $t$  is an unknown constant, are perpendicular. Find the value of  $t$ . **(3 marks)**

- 5** Fill in the multiple blanks:  
Given that  $\mathbf{a} = (5, 1, -1)$  and  $\mathbf{b} = (2, 3, 4)$ , calculate the components of  $(\mathbf{a} + 3\mathbf{b}) \times (\mathbf{a} - 2\mathbf{b})$ .  
 $(\mathbf{a} + 3\mathbf{b}) \times (\mathbf{a} - 2\mathbf{b}) = ([Blank1], [Blank2], [Blank3])$ . **(6 marks)**

6 The derivative with respect to  $x$  of  $y = 3x^3 \ln 2x$  is

•

$$\frac{dy}{dx} = 9x^2 \ln 2x + 3x^2 \ln 2x$$

•

$$\frac{dy}{dx} = 3x^2(3 \ln 2x + 1)$$

Correct answer

•

$$\frac{dy}{dx} = 9x^3 + 3x^2 \ln 2x$$

•

$$\frac{dy}{dx} = 9x^2(\ln 2x + 1)$$

**(5 marks)**

7 The derivative with respect to  $x$  of  $y = e^{6x} \sin(9x)$  is

•

$$\frac{dy}{dx} = \frac{1}{6}e^{6x} \cos(9x)$$

•

$$\frac{dy}{dx} = e^{6x}(6 \sin(9x) + 9 \cos(9x))$$

•

$$\frac{dy}{dx} = e^{6x}(6 \sin(9x) - 9 \cos(9x))$$

•

$$\frac{dy}{dx} = 54e^{6x} \cos(9x)$$

**(5 marks)**

8 By reversing the order of integration evaluate

$$\int_0^{\sqrt{\pi}} \int_y^{\sqrt{\pi}} \cos(x^2) \, dx dy.$$

**(7 marks)**

**9** By using an appropriate substitution we can write

$$\int \sin^3(x) \cos^n(x) dx = \int u^9 - u^7 du.$$

What is the value of the integer  $n$ ?

**(6 marks)**

**10** The eigenvalues of the matrix

$$\begin{pmatrix} 6 & -11 & 18 \\ 0 & -18 & 23 \\ 0 & 0 & 37 \end{pmatrix}$$

are

- -6, 18, -37
- 6, -18, 37
- 6, -11, 18
- 18, 23, 37

**(4 marks)**

**11** If

$$\begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix}$$

is an eigenvector of

$$\begin{pmatrix} -2 & -4 & 2 \\ -2 & 1 & 2 \\ 4 & 2 & 5 \end{pmatrix}$$

what is its corresponding eigenvalue?

**(6 marks)**

**12** Matrix  $A$  is a  $3 \times 4$  matrix. Matrix  $B$  is a  $3 \times 3$  matrix. Which of the following matrix expressions are defined? Select all that apply.

- $BA^2$
- $BA - 2A$
- $A + 2B$
- $B^2 - AB$

**(4 marks)**

- 13** The matrix  $A$  is given by

$$\begin{pmatrix} 3 & k \\ -4 & -3 \end{pmatrix}$$

Find the value of  $k$  for which  $A = A^{-1}$ , where  $A^{-1}$  is the inverse of  $A$ . **(6 marks)**

- 14** Fill in the multiple blanks: The first four terms, given in order, of a geometric sequence are  $a, b, c, d$  and the first four terms, given in order, of an arithmetic sequence are  $a, \frac{b}{2}, \frac{c}{4}, d - 70$ , where  $a, b, c, d$  are real numbers. The value of the common ratio,  $r$ , of the geometric sequence is [Blank1] and the value of  $a$  is [Blank2]. **(8 marks)**

**End of Question Paper**