The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **Spring Semester 2020–2021**

Time Series **2 hours and 30 minutes**

This is an open book exam.

Answer ***both*** *questions.*

You can work on the exam during the 24 hour period starting from 10am (BST), and you must submit your work within ***2 hours and 30 minutes*** *of accessing the exam paper or by the end of the 24 hour period (whichever is earlier).*

Late submission will not be considered without extenuating circumstances. *Calculations should be performed by hand. A university-approved calculator may be used. The use of any other calculational device, software or service is not permitted. To gain full marks, you will need to show your working.*

By uploading your solutions you declare that your submission consists entirely of your own work, that any use of sources or tools other than material provided for this module is cited and acknowledged, and that no unfair means have been used.

1 (i) Given the following process

$$y_t = \sum_{i=0}^{3} (-1)^i \beta_i \epsilon_{t-i}$$

where $i = 0, 1, 2, 3$, β_i denote the parameters and the ϵ_ts are white noise $\mathrm{WN}(0, \sigma^2)$:

(a) Obtain all corresponding non-zero values of the covariance function γ_h. ***(15 marks)***

(b) Obtain the autocorrelation function (ACF) of the process. ***(10 marks)***

(ii) Consider 300 observations of a time series $\{y_t\}$ with ACF and partial autocorrelation function (PACF) given in the following tables.

Lag (h)	1	2	3	4
ACF (r_h)	A	B	0.260	0.172
PACF ($\alpha_h^{(h)}$)	0.698	-0.122	0.023	0.023

(a) Find the missing values of A and B. ***(10 marks)***

(b) Test whether $\{y_t\}$ is consistent with moving average models: MA(1) and MA(2). ***(9 marks)***

(c) Test whether $\{y_t\}$ is consistent with autoregressive models: AR(1) and AR(2). ***(6 marks)***

2 A model is developed to determine average industry-level wage rates during a recessionary period. Accordingly, current wage rates w_t at time t are linked to the previous period wage rates w_{t-1} and a margin μ_t via the following rule:

$$w_t = w_{t-1} + \mu_t$$

A time series model for the margin μ_t is adopted given by

$$\mu_t = 0.7\mu_{t-1} + \lambda_t$$

where, λ_t is a white noise innovation following a normal distribution with variance equal to 9. Define the state vector

$$\beta_t = \begin{bmatrix} w_{t-1} \\ \mu_t \end{bmatrix}.$$

(i) Find a state space model for the current period wage rate w_t, i.e.

$$w_t = x_t^\top \beta_t + \epsilon_t$$
$$\beta_t = F\beta_{t-1} + \zeta_t$$

Determine the elements of x_t, F and, provide the distributions of ϵ_t and ζ_t. ***(12 marks)***

(ii) After 300 days the posterior distribution of β_{300} given data $w_{1:300} = \{w_1, w_2, ..., w_{300}\}$ is:

$$\beta_{300} \mid w_{1:300} \sim N\left\{\begin{bmatrix} 4 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 4 \end{bmatrix}\right\}.$$

Provide the 3-step ahead forecast distribution of w_{303} given data $w_1, w_2, ..., w_{300}$. ***(23 marks)***

(iii) Modify the transition matrix F of the model in part (i), such that the previous period wage rate w_{t-1} follows a linear function in time (t), plus a linear combination of random errors. Obtain the corresponding expression linking w_{t-1} to (t). ***(15 marks)***

End of Question Paper