The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **Spring Semester 2020–2021**

Further General Engineering Mathematics **3 hr 30 mins**

This is an open book exam.

Answer all questions.

You can work on the exam during the 24 hour period starting from 10am (BST), and you must submit your work within 3 hours and 30 minutes of accessing the exam paper or by the end of the 24 hour period (whichever is earlier).

Late submission will not be considered without extenuating circumstances. *Calculations should be performed by hand. A university-approved calculator may be used. The use of any other calculational device, software or service is not permitted. To gain full marks, you will need to show your working.*

By uploading your solutions you declare that your submission consists entirely of your own work, that any use of sources or tools other than material provided for this module is cited and acknowledged, and that no unfair means have been used.

1 Find and classify *all* the critical points of the function

$$f(x,y) = x^3 + 2y^3 - 3x - 6y.$$

(10 marks)

2 Find the Fourier cosine series of the function $f : [0,\pi) \to \mathbb{R}$ defined on $[0,\pi)$ by

$$f(t) = t - H(t-1).$$

(10 marks)

3 The charge $y(t)$ of a current in a circuit is described by

$$y''(t) + 2y'(t) + 5y(t) \; = \; 1,$$

subject to the initial conditions $y(0) = y'(0) = 0$. Use the Laplace transform to determine the charge $y(t)$ at time $t > 0$.
Remark: Your answer must use the Laplace transform and should clearly indicate where standard transforms and properties of the transform are used.

(10 marks)

4 Let $T \subset \mathbb{R}^2$ be the region bounded by the lines $y = x$, $y = -x$, and $x = \sqrt{\pi}$, and let $f(x,y) = x^2 \cos(xy)$. Find

$$\iint_T f(x,y)dA.$$

(10 marks)

5 The temperature $T(x,t)$ in a uniform metal rod of length π satisfies the heat equation

$$\frac{\partial T}{\partial t} \; = \; \frac{\partial^2 T}{\partial x^2}. \qquad (\star)$$

(i) Show that the function $C\cos(nx)e^{-n^2 t}$ is a solution to $(\star)$ when C is an arbitrary constant. **(4 marks)**

(ii) Suppose that boundary conditions are given on $T(x,t)$ which lead to the general solution

$$T(x,t) \; = \; A_0 + \sum_{n=1}^{\infty} A_n \cos(nx)e^{-n^2 t},$$

where $A_0, A_1, \ldots$ are arbitrary constants. Determine the A_n coefficients and write down the general solution if $T(x,t)$ satisfies the initial condition $T(x,0) = x$. Simplify your answer as much as possible. **(6 marks)**

End of Question Paper