Data provided:
Graph Paper, Laplace Transform Table

MAS312

The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS — Autumn Semester 2011–2012

CLASSICAL CONTROL THEORY — 2 hours

*Marks will be awarded for your best **four** answers.*

1 (i) The output, $y(t)$, for $t \geq 0$, when the input is a step input and the initial conditions are $y(0) = \dot{y}(0) = 0$ for the linear control system with transfer function

$$H(s) = \frac{s+3}{(s^2+s+2)(s+2)^2}$$

has the form

$$y(t) = A + Be^{-2t} + Cte^{-2t} + e^{-\alpha t}\left(D\cos(\beta t) + E\sin(\beta t)\right)$$

Calculate the values of A, B, C, α and β in the formula, and obtain mathematical expressions for D and E.
You do not need to calculate the numerical values of D and E. ***(8 marks)***

(ii) Consider the linear control system with transfer function

$$G(s) = \frac{s}{s^2+7s+12}$$

(a) Find the impulse response in the time domain for $t \geq 0$ for zero initial conditions.

(b) For zero initial conditions determine the response to the sinusoidal input $A\sin(2t)$ for t tending to infinity. ***(7 marks)***

(iii) Prove that

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$$

A system has the output Laplace transform

$$Y(s) = \frac{K+11s}{K+12s+s^2+5s^3+s^4}R(s) + \frac{1}{K+12s+s^2+5s^3+s^4}D(s)$$

where $R(s)$ is the Laplace transform of the input and $D(s)$ is the Laplace transform of a disturbance.

(a) For a unit step input and zero disturbance determine the steady state output error.

(b) For zero input and unit step disturbance determine the steady state output response. ***(10 marks)***

2 (i) Find the closed-loop transfer function $H(s) = \dfrac{Y(s)}{R(s)}$ of the *unity* negative feedback control system with output $Y(s)$, input $R(s)$ and feedforward transfer function $C(s)G(s)$ in the forward path.

If $G(s) = \dfrac{n(s)}{d(s)}$ and $C(s) = \dfrac{n_c(s)}{d_c(s)}$, give the equations satisfied by the open-loop and closed-loop zeros and poles.

(5 marks)

If $C(s) = ks + \dfrac{1}{s} + 2$ in the above control system, give the differential equation satisfied by the input, $v(t)$, and output, $w(t)$, of the linear subsystem $C(s)$.

Suppose that

$$G(s) = \frac{1}{s^2 - 2s + 5}$$

Use the Routh-Hurwitz method to establish the range of gains k for which the closed-loop system with feedforward transfer function $C(s)G(s)$ is stable.

(8 marks)

(ii) A system has characteristic equation

$$s^2 + 10s + 3 + k = 0$$

Using the Routh Table establish the values of the gain parameter k which yield poles with real parts less than -4. ***(6 marks)***

(iii) Determine the number of stable and unstable roots of the polynomial

$$p(s) = 2s^3 + 4s^2 + 4s + 12$$

(6 marks)

3 (i) Sketch the Root Locus plot of the constant-gain feedback system with

$$G(s) = \frac{s+3}{(s^2 + 2s + 5)(s^2 + 5s + 4)}$$

ensuring that you compute any angles of departure and all crossover points. Give the range of gains for closed-loop stability.

(20 marks)

(ii) Using the elementary properties of the Root Locus, justify the statement: *"If the transfer function $G(s)$ has a single unstable real zero and a simple pole at 0, and all other poles are stable, then it is not stabilizable by constant gain feedback."*

(5 marks)

4 You are asked to design a constant-gain control for the unstable second-order system

$$G(s) = \frac{s+1}{(s-1)^2+1}$$

which has the following closed-loop specifications:

- The 2% settling time is no more than 10 seconds.
- The overshoot should not exceed 5%.

(i) Give the relations between settling time and the real part of the pole pair, and between overshoot and the damping ratio ζ. Sketch the region in the complex plane that must contain the closed-loop poles if the above specifications are to be met.

(13 marks)

(ii) Sketch the Root Locus and hence obtain a control gain that achieves the specifications.

(12 marks)

5 (i) Sketch the Nyquist plot (for $\omega > 0$) for

$$G(s) = \frac{20(s+1)}{(s+5)^2+1}$$

Find all crossings of the real and imaginary axes.

(12 marks)

(ii) State the general Nyquist Stability Criterion.

Sketch the Nyquist contour (for $\omega > 0$) for

$$G(s) = \frac{s+5}{s(s+1)(s+2)}$$

and find all crossings of the real axis. Find the values of the gain k for which the closed-loop system (in the constant-gain configuration) is stable.

(13 marks)

Table of Laplace Transform Pairs

Time Function	Laplace Transform
$h(t)$	$\frac{1}{s}$
$\frac{t^n}{n!}$	$\frac{1}{s^{n+1}}$
e^{-at}	$\frac{1}{s+a}$
$\frac{t^n e^{-at}}{n!}$	$\frac{1}{(s+a)^{n+1}}$
$\cos\omega_0 t$	$\frac{s}{s^2+\omega_0^2}$
$\sin\omega_0 t$	$\frac{\omega_0}{s^2+\omega_0^2}$
$e^{-at}\cos\omega_0 t$	$\frac{s+a}{(s+a)^2+\omega_0^2}$
$e^{-at}\sin\omega_0 t$	$\frac{\omega_0}{(s+a)^2+\omega_0^2}$

End of Question Paper