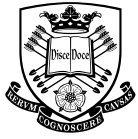


Data provided:
Graph Paper, Laplace Transform Table



The
University
Of
Sheffield.

MAS312

SCHOOL OF MATHEMATICS AND STATISTICS

Autumn Semester 2011–2012

CLASSICAL CONTROL THEORY

2 hours

*Marks will be awarded for your best **four** answers.*

- 1 (i) The output, $y(t)$, for $t \geq 0$, when the input is a step input and the initial conditions are $y(0) = \dot{y}(0) = 0$ for the linear control system with transfer function

$$H(s) = \frac{s + 3}{(s^2 + s + 2)(s + 2)^2}$$

has the form

$$y(t) = A + Be^{-2t} + Cte^{-2t} + e^{-\alpha t} (D \cos(\beta t) + E \sin(\beta t))$$

Calculate the values of A , B , C , α and β in the formula, and obtain mathematical expressions for D and E .

You do not need to calculate the numerical values of D and E . **(8 marks)**

- (ii) Consider the linear control system with transfer function

$$G(s) = \frac{s}{s^2 + 7s + 12}$$

- (a) Find the impulse response in the time domain for $t \geq 0$ for zero initial conditions.
- (b) For zero initial conditions determine the response to the sinusoidal input $A \sin(2t)$ for t tending to infinity. **(7 marks)**

- (iii) Prove that

$$\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} sF(s)$$

A system has the output Laplace transform

$$Y(s) = \frac{K + 11s}{K + 12s + s^2 + 5s^3 + s^4} R(s) + \frac{1}{K + 12s + s^2 + 5s^3 + s^4} D(s)$$

where $R(s)$ is the Laplace transform of the input and $D(s)$ is the Laplace transform of a disturbance.

- (a) For a unit step input and zero disturbance determine the steady state output error.
- (b) For zero input and unit step disturbance determine the steady state output response. **(10 marks)**

- 2 (i) Find the closed-loop transfer function $H(s) = \frac{Y(s)}{R(s)}$ of the *unity* negative feedback control system with output $Y(s)$, input $R(s)$ and feedforward transfer function $C(s)G(s)$ in the forward path.

If $G(s) = \frac{n(s)}{d(s)}$ and $C(s) = \frac{n_c(s)}{d_c(s)}$, give the equations satisfied by the open-loop and closed-loop zeros and poles.

(5 marks)

If $C(s) = ks + \frac{1}{s} + 2$ in the above control system, give the differential equation satisfied by the input, $v(t)$, and output, $w(t)$, of the linear sub-system $C(s)$.

Suppose that

$$G(s) = \frac{1}{s^2 - 2s + 5}$$

Use the Routh-Hurwitz method to establish the range of gains k for which the closed-loop system with feedforward transfer function $C(s)G(s)$ is stable.

(8 marks)

- (ii) A system has characteristic equation

$$s^2 + 10s + 3 + k = 0$$

Using the Routh Table establish the values of the gain parameter k which yield poles with real parts less than -4 .

(6 marks)

- (iii) Determine the number of stable and unstable roots of the polynomial

$$p(s) = 2s^3 + 4s^2 + 4s + 12$$

(6 marks)

- 3 (i) Sketch the Root Locus plot of the constant-gain feedback system with

$$G(s) = \frac{s + 3}{(s^2 + 2s + 5)(s^2 + 5s + 4)}$$

ensuring that you compute any angles of departure and all crossover points. Give the range of gains for closed-loop stability.

(20 marks)

- (ii) Using the elementary properties of the Root Locus, justify the statement: "If the transfer function $G(s)$ has a single unstable real zero and a simple pole at 0 , and all other poles are stable, then it is not stabilizable by constant gain feedback."

(5 marks)

- 4 You are asked to design a constant-gain control for the unstable second-order system

$$G(s) = \frac{s + 1}{(s - 1)^2 + 1}$$

which has the following closed-loop specifications:

- The 2% settling time is no more than 10 seconds.
 - The overshoot should not exceed 5%.
- (i) Give the relations between settling time and the real part of the pole pair, and between overshoot and the damping ratio ζ . Sketch the region in the complex plane that must contain the closed-loop poles if the above specifications are to be met.

(13 marks)

- (ii) Sketch the Root Locus and hence obtain a control gain that achieves the specifications.

(12 marks)

- 5 (i) Sketch the Nyquist plot (for $\omega > 0$) for

$$G(s) = \frac{20(s + 1)}{(s + 5)^2 + 1}$$

Find all crossings of the real and imaginary axes.

(12 marks)

- (ii) State the general Nyquist Stability Criterion.

Sketch the Nyquist contour (for $\omega > 0$) for

$$G(s) = \frac{s + 5}{s(s + 1)(s + 2)}$$

and find all crossings of the real axis. Find the values of the gain k for which the closed-loop system (in the constant-gain configuration) is stable.

(13 marks)

Table of Laplace Transform Pairs

Time Function	Laplace Transform
$h(t)$	$\frac{1}{s}$
$\frac{t^n}{n!}$	$\frac{1}{s^{n+1}}$
e^{-at}	$\frac{1}{s+a}$
$\frac{t^n e^{-at}}{n!}$	$\frac{1}{(s+a)^{n+1}}$
$\cos \omega_0 t$	$\frac{s}{s^2 + \omega_0^2}$
$\sin \omega_0 t$	$\frac{\omega_0}{s^2 + \omega_0^2}$
$e^{-at} \cos \omega_0 t$	$\frac{s+a}{(s+a)^2 + \omega_0^2}$
$e^{-at} \sin \omega_0 t$	$\frac{\omega_0}{(s+a)^2 + \omega_0^2}$

End of Question Paper